Washington Experimental Mathematics Lab

Number Theory and Noise

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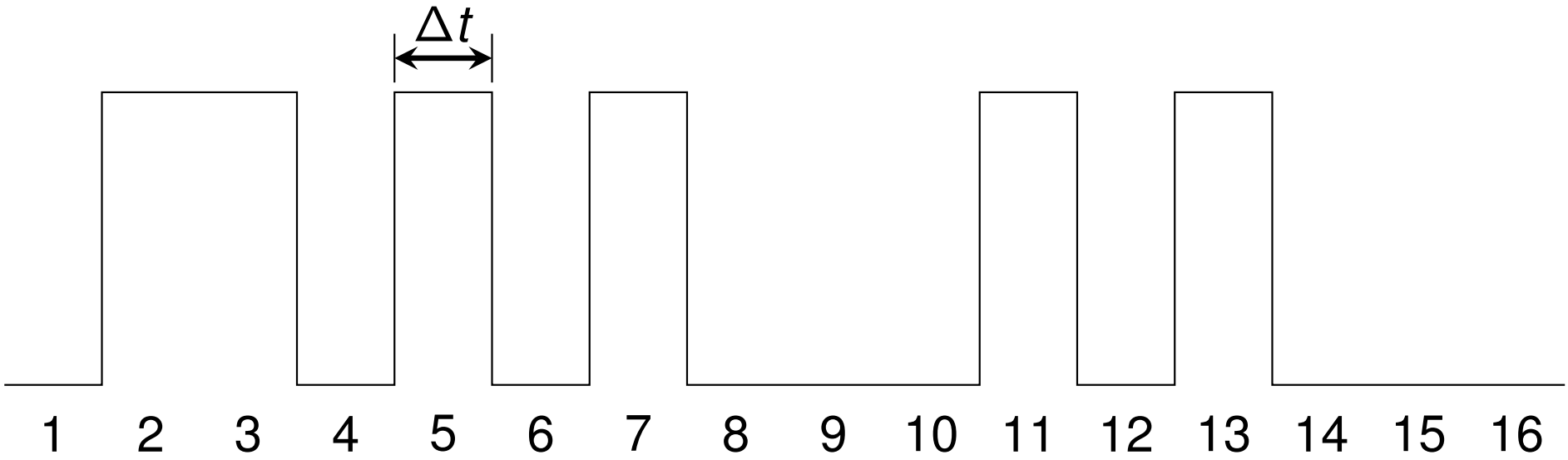
Number Theory and Noise

This project investigates the possibilities arising from representing sets of positive integers as sound.

A digital audio file is created from a given set A of positive integers by setting sample number i to a non-zero constant c for all i in the set. All other samples are set to zero.

Number Theory and Noise

For example, the waveform for the primes starts like this:

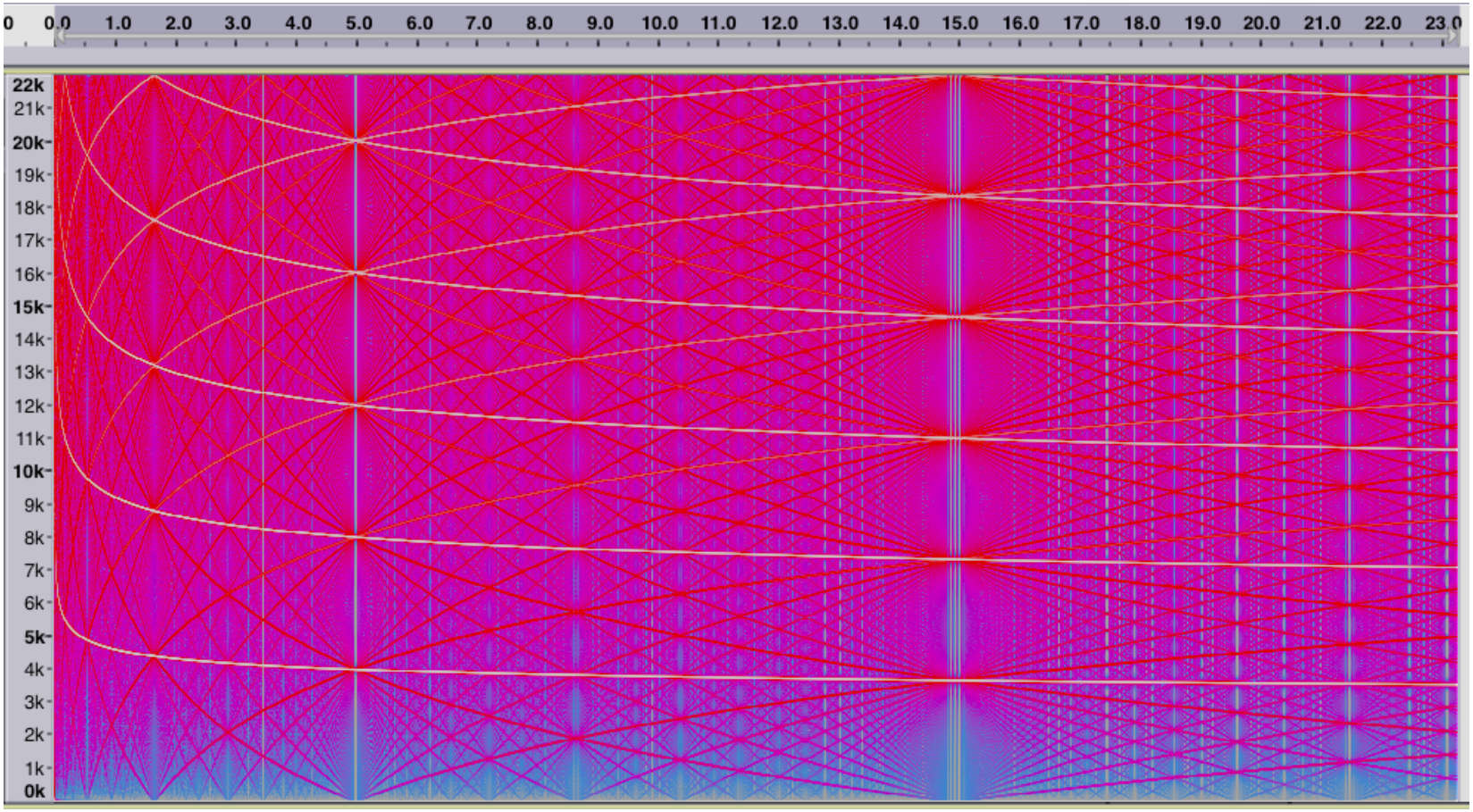


We use the standard CD-audio sampling rate of 44100 samples per second, so $\Delta t = \frac{1}{44100} = 0.0000226757...$ seconds.
For many sets, the result is what most people would describe as *noise*.

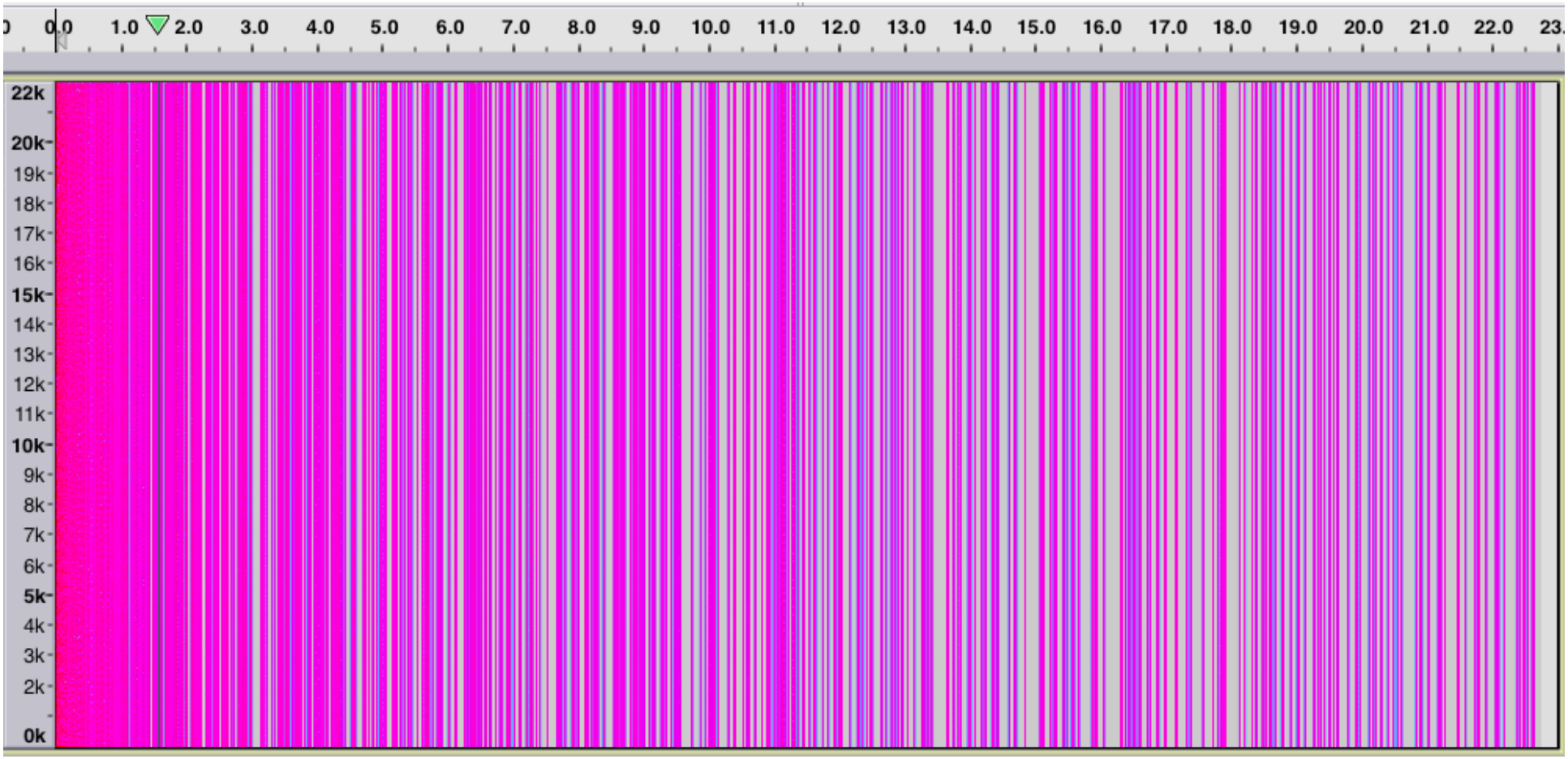
Issues

- Working with large primes
- Sequences, like quadratics, grow too fast
- Computing limitations (e.g. numbers n such that $(2^n + 1)/3$ is prime)

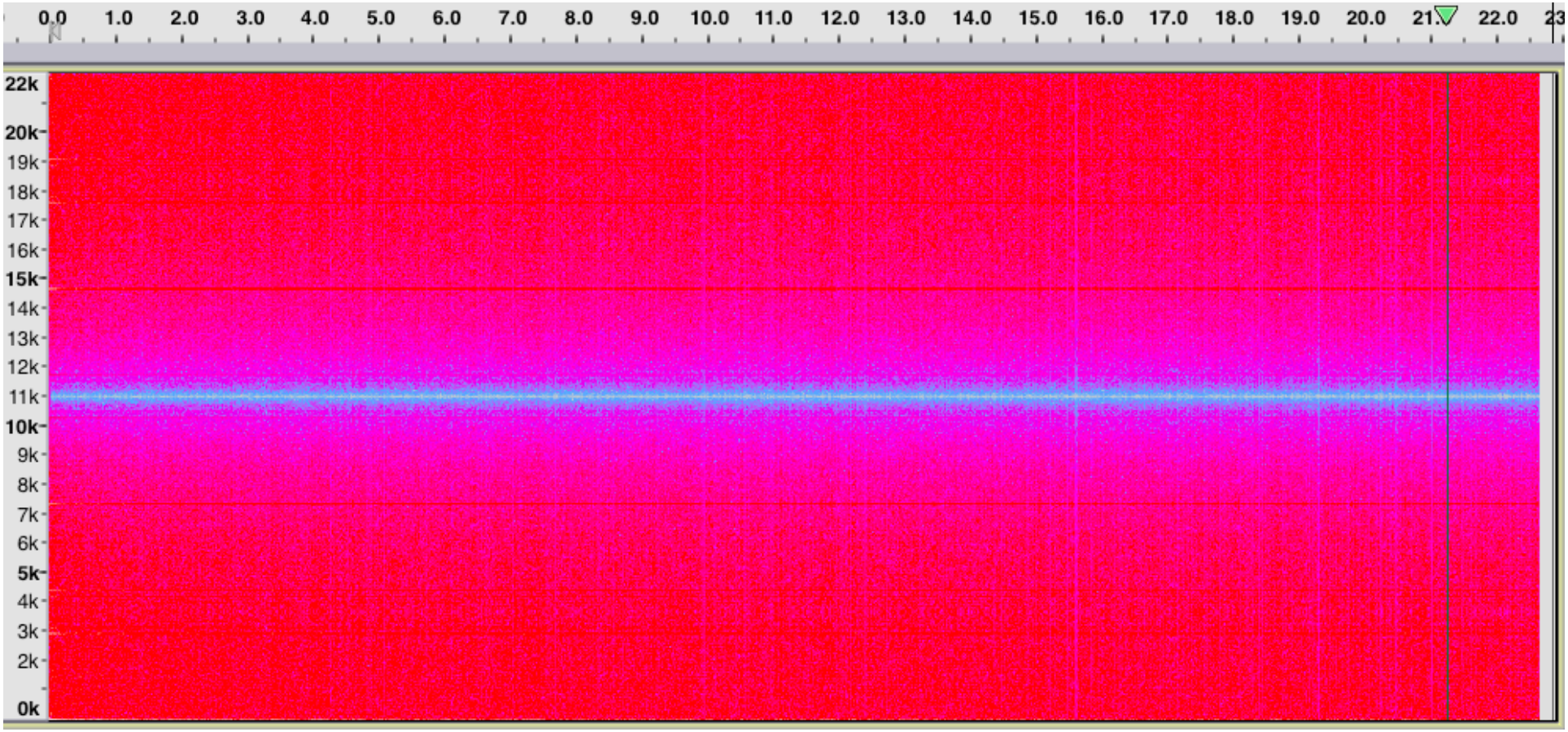
$a_n = \lfloor n \log(n) \rfloor$



A000879: number of primes < the $(n^{th} prime)^2$



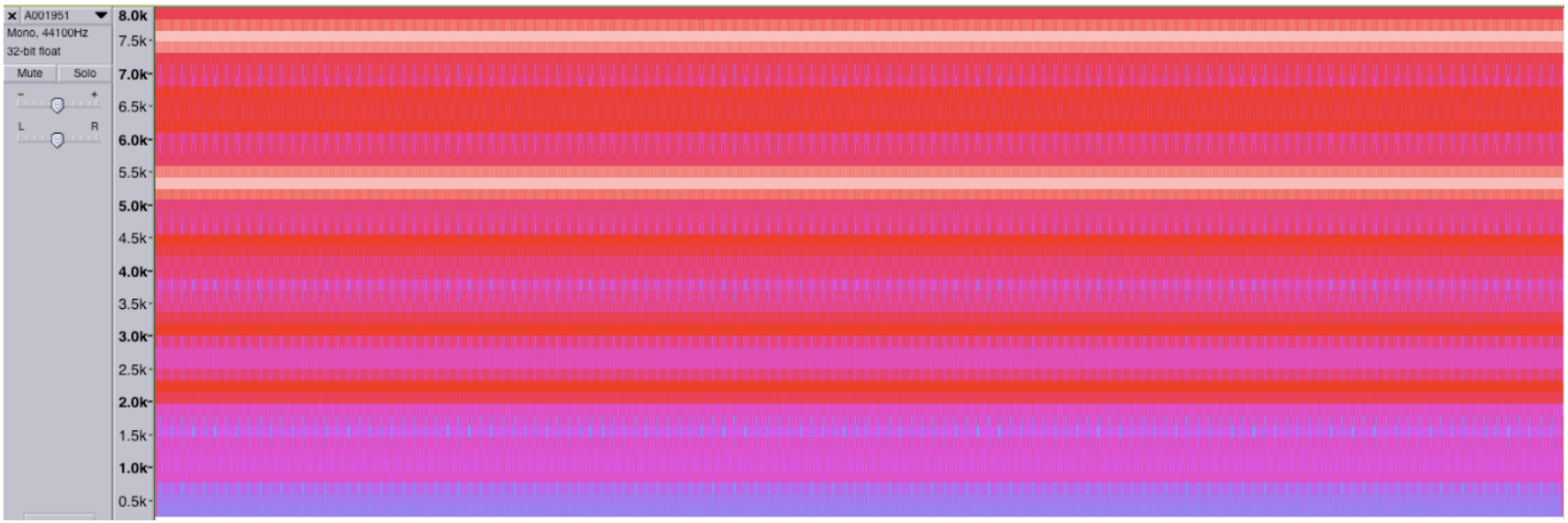
A001097: Twin Primes



A001951: Beatty Sequence of $\sqrt{2}$

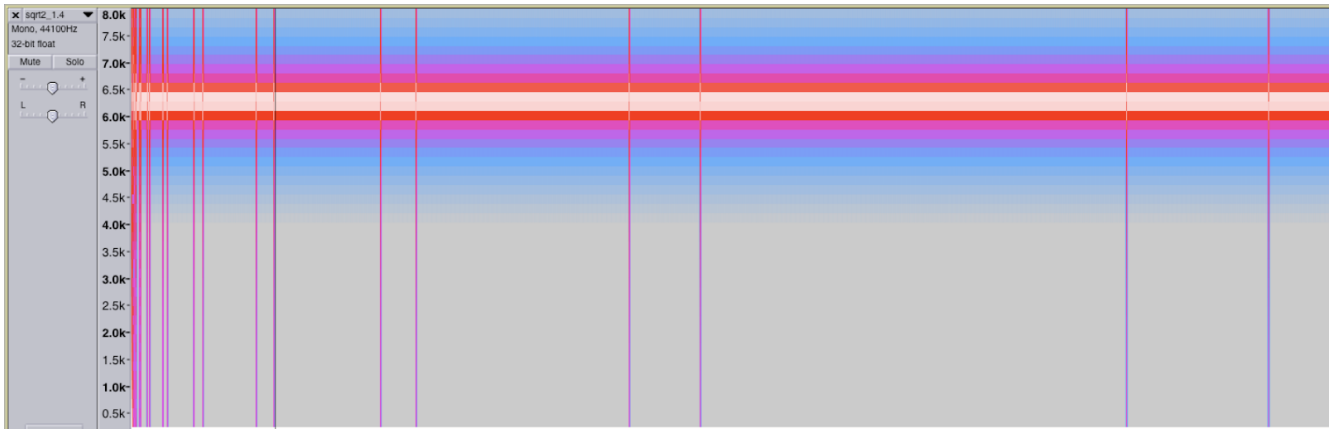
a Beatty sequence is the sequence of integers found by taking the floor of the positive multiples of a positive irrational number.

$$a_n = \left\lfloor n\sqrt{2} \right\rfloor$$

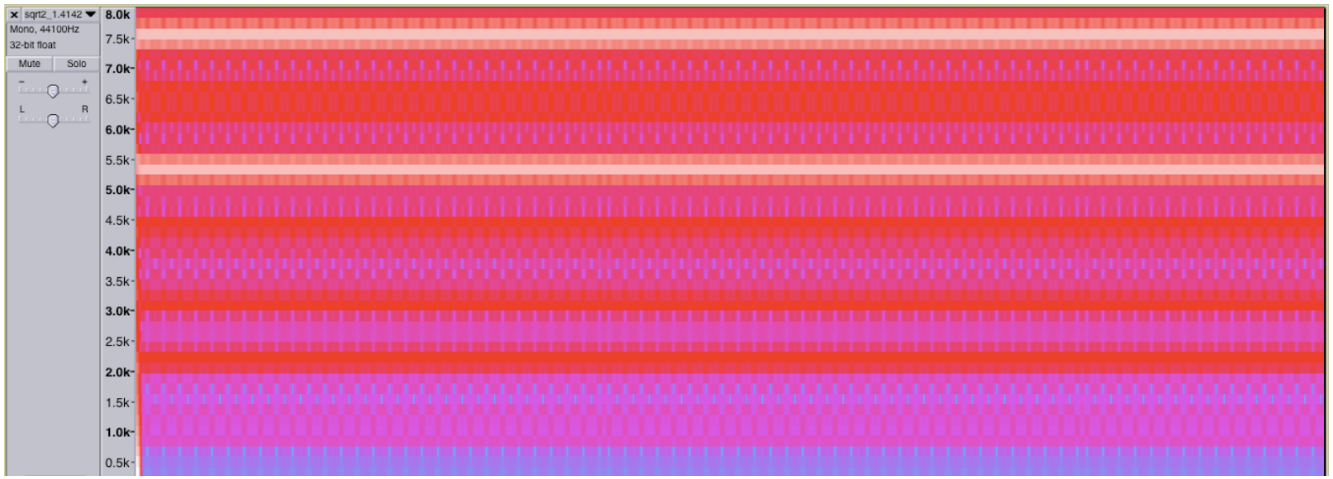


Using Decimal Approximation of $\sqrt{2}$

$\sqrt{2}$ as 1.4



$\sqrt{2}$ as 1.4142



Future goals

- Finding sequences to focus on
- Understanding $\lfloor n \log(n) \rfloor$
- Understanding Beatty sequences with different approximations of the irrational part