Washington Experimental Mathematics Lab
Randomly Mixing Fluids

Department of Mathematics
University of Washington
Keith Fife, Yiqi Huang
Max Goering
Soumik Pal

Autumn 2017
Gather and Spread
Gather and Spread
Advection: \((x, y, z) \mapsto (x, y, z) + (s_x(y), s_y(x), 0)\)

Diffusion: \((x, y, z) \mapsto \frac{1}{4}((x - 1, y, z_1) + (x + 1, y, z_2) + (x, y - 1, z_3) + (x, y + 1, z_4))\)
Gather and Spread

![Graph showing the relationship between number of experiments and number of points. The x-axis represents the number of points ranging from 60 to 200, and the y-axis represents the number of experiments ranging from 0 to 1600. The graph shows a trend where the number of experiments increases with the number of points.]
Kendall’s $\tau$ Statistic:

Given a datum $(x_i, y_i)$, calculate the permutation of $x_i$ from left to right compared to the initial arrangement in line, Kendall’s $\tau$ is the count of inversions. IE, the number of $(i, j)$ such that $i$ is to left left of $j$ initially, but after the final time $i$ is to the right of $j$. 
Toroidal Mixing

\[ N: \text{ The total number of lattice points.} \]

Standard Deviation: \[ \sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \frac{1}{N})^2} \]

Entropy: \[ s = -\sum_{i=1}^{N} x_i \ln x_i \]

Maximum Entropy: \[ s_{max} = -\sum_{i=1}^{N} \frac{1}{N} \ln \frac{1}{N} \]

Relative Entropy: \[ \Delta s = s_{max} - s = \ln N + \sum_{i=1}^{N} x_i \ln x_i \]
Mixing Plots

Mixing Steps: 250
Number of Simulations: 500
Ending Statistics Distributions

Average of Deviation Distribution: $4.4 \cdot 10^{-5}$
Standard Deviation of Deviation Distribution: $2.1 \cdot 10^{-5}$
Average of Entropy Distribution: $1.1 \cdot 10^{-1}$
Standard Deviation of Entropy Distribution: $1.2 \cdot 10^{-2}$
Exponential Decay Measurement

Deviation Regression Slope: \(-1.19 \cdot 10^{-2}\)
Deviation Regression \(R^2\): 99.3%
Entropy Regression Slope: \(-1.68 \cdot 10^{-2}\)
Entropy Regression \(R^2\): 99.6%
Comparison Against Diffusion Only

**Method:** Diffusion Only – Mixing Model
Comparison Against Diffusion Only

Method: \(\text{Diffusion Only} - \text{Mixing Model} / \text{Diffusion Only}\)
Questions?