Washington Experimental Mathematics Lab

Randomly Mixing Fluids

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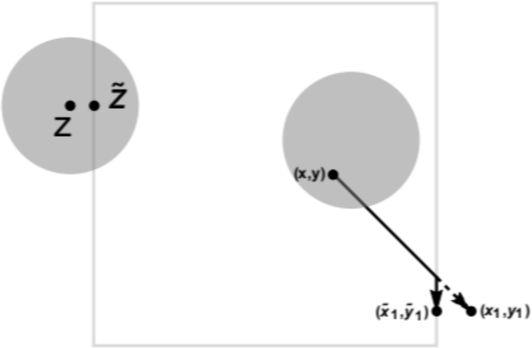
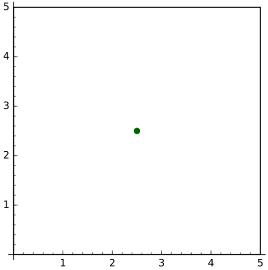
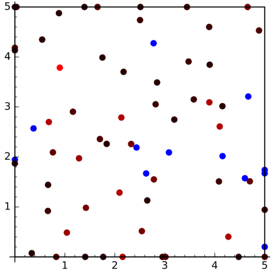
Autumn 2017

Gather and Spread

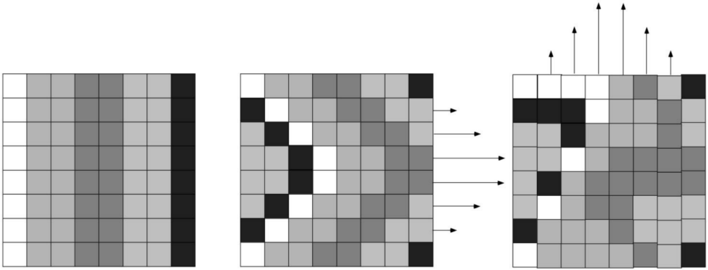


Gather and Spread





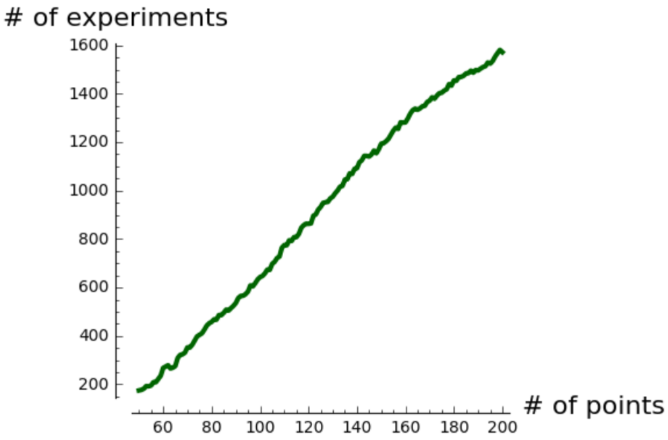
Toroidal Lattice Mixing



Advection: $(x, y, z) \mapsto (x, y, z) + (s_x(y), s_y(x), 0)$

Diffusion: $(x, y, z) \mapsto$
$\frac{1}{4}((x-1, y, z_1)+(x+1, y, z_2)+(x, y-1, z_3)+(x, y+1, z_4))$

Gather and Spread



Kendall's τ

Kendall's τ Statitistic:

Given a datum (x_i, y_i), calculate the permutation of x_i from left to right compared to the initial arrangement in line, Kendall's τ is the count of inversions.
IE, the number of (i, j) such that i is to left left of j initially, but after the final time i is to the right of j.

Toroidal Mixing

N: The total number of lattice points.

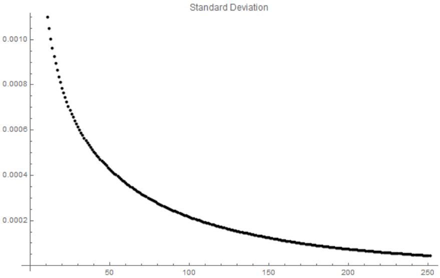
Standard Deviation: $\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \frac{1}{N})^2}$

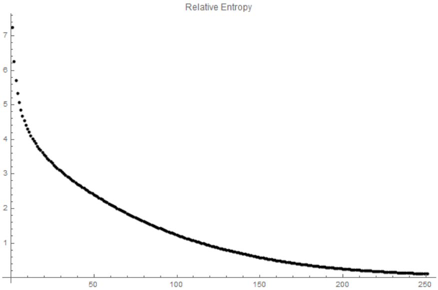
Entropy: $s = -\sum_{i=1}^{N} x_i \ln x_i$

Maximum Entropy: $s_{max} = -\sum_{i=1}^{N} \frac{1}{N} \ln \frac{1}{N}$

Relative Entropy: $\Delta s = s_{max} - s = \ln N + \sum_{i=1}^{N} x_i \ln x_i$

Mixing Plots

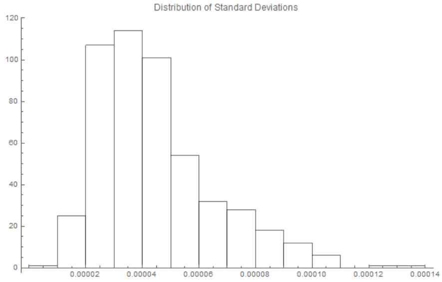


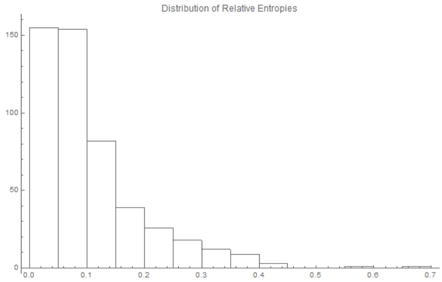


Mixing Steps: 250

Number of Simulations: 500

Ending Statistics Distributions





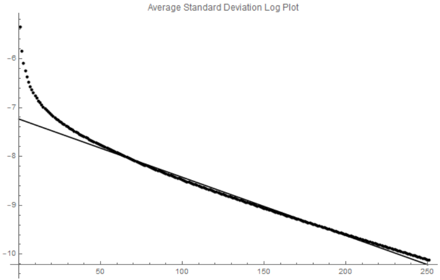
Average of Deviation Distribution: $4.4 \cdot 10^{-5}$

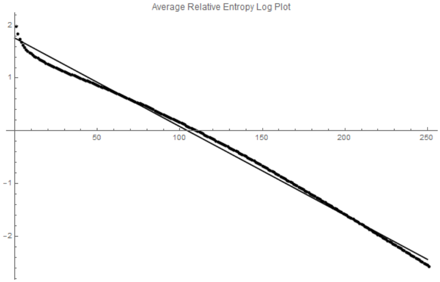
Standard Deviation of Deviation Distribution: $2.1 \cdot 10^{-5}$

Average of Entropy Distribution: $1.1 \cdot 10^{-1}$

Standard Deviation of Entropy Distribution: $1.2 \cdot 10^{-2}$

Exponential Decay Measurement





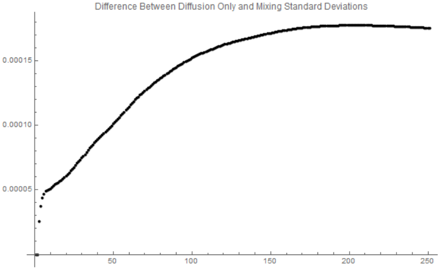
Deviation Regression Slope: $-1.19 \cdot 10^{-2}$

Deviation Regression R^2: 99.3%

Entropy Regression Slope: $-1.68 \cdot 10^{-2}$

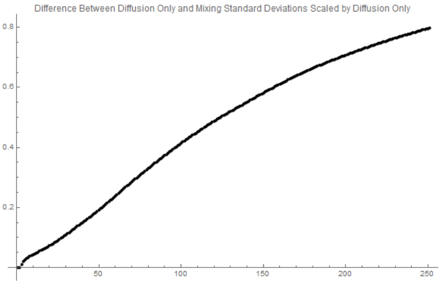
Entropy Regression R^2: 99.6%

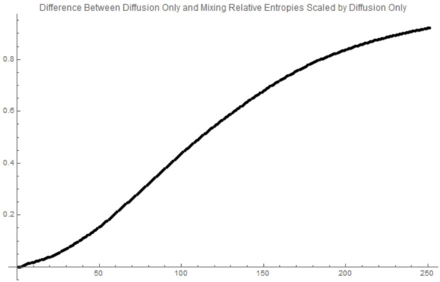
Comparison Against Diffusion Only



Method: Diffusion Only – Mixing Model

Comparison Against Diffusion Only





Method: $\frac{\text{Diffusion Only} - \text{Mixing Model}}{\text{Diffusion Only}}$

Questions

Questions?