Parabolic Double Cosets in Symmetric Groups

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Spring 2018

Permutations

- A *permutation* is a rearrangement of things.
- $\{$stop, spot, tops, pots, post$\}$ are permutations of the letters o,p,s,t.
- The following are **all** permutations of $\{1, 2, 3\}$:

[123]	[213]
[132]	[312]
[231]	[321]

- We call this set of $3! = 6$ permutations the *symmetric group* on 3 elements, and we denote it by S_3.

Parabolic cosets

- Which permutations can you make from [43172864] by only swapping numbers of the same color?
- Which permutations can you make from [43172864] by only swapping numbers underlined by the same color?
- The set of all such permutations are examples of left/right *parabolic cosets*.
- What if we allow both types of swapping?

Parabolic double cosets

- If we allow swapping of certain adjacent values **and** certain adjacent positions, we end up with a *parabolic double coset*.

- Example: $\{[23514], [24513], [25413], [25314]\}$ is a parabolic double coset in S_5.

Questions

- How many parabolic double cosets are in S_n?
- There are 19 parabolic double cosets in S_3, 5597524 in S_8, and 34300156146805 in S_{13}. We have yet to compute this number for S_{14}.
- What are the most efficient ways of counting them?

Strategies

- Counting them by their “smallest" element.
- Taking advantage of symmetries in the group.
- Counting them by their structure under Bruhat order.

Partial orders

Examples: