Impartial Games

- 2 players with same rules
- Finite steps
- No random chance
- No secrets
- Whoever goes last wins
Example - Nim

- Most well-known impartial game
- Example

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Example - Nim

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![Diagram of a Nim game setup]
Example - Nim

- Most well-known impartial game
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Example - Nim

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Example - Nim

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- Example
Example - Nim

• Most well-known impartial game

Example

Grundy-Sprague theorem: every game with the qualities on the previous page is equivalent to some game of Nim.

• The **nimber** of a game tells you what it’s equivalent to.
Rook Placement Game

- Start with a board $B$, a finite collection of cells on a grid.
- Two players take turns placing rooks on $B$ so that no two rooks *attack* each other.
- Whoever places the last rook wins.

Question: Given the board $B$, who wins? What’s the nimber?
Example game
Example game
Example game
Example game

Purple wins!
Rectangular boards are boring: If $B$ is an $m \times n$ board with $m \leq n$, then player 1 wins if $m$ is odd, and player 2 wins if $m$ is even.

What if there are holes in the board?
Theorem

Let $B$ be an $m \times n$ rectangular board with $m \leq n$, and let $B'$ be a board obtained from $B$ by removing at most $n - 2$ cells if $m$ is even, and $n - 1$ cells if $m$ is odd. Then $B$ and $B'$ will have the same winner.
Future Goals

Next steps
- Find all placements of the minimum number of holes to change the winner for rectangular boards.
- Staircase boards
- Other boards

Challenges
- Not obvious what a good move is
- Computational Complexity