Washington Experimental Mathematics Lab

Orbit Structure of Crystal Operators

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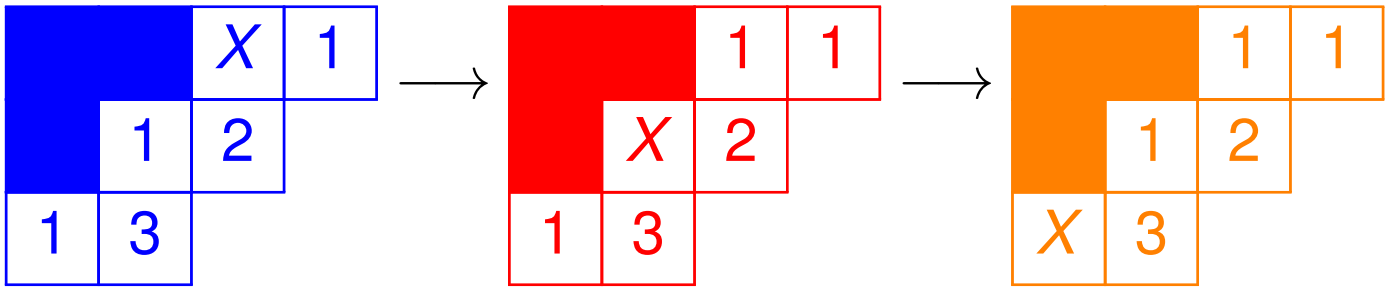
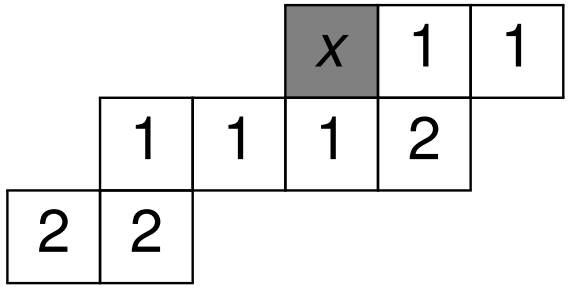
Review

Orbit:

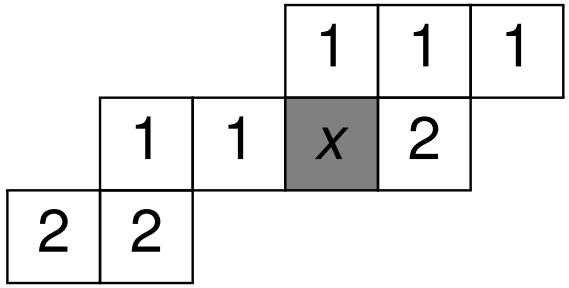


Review

What is a Jump?



↓ no jumps



Review

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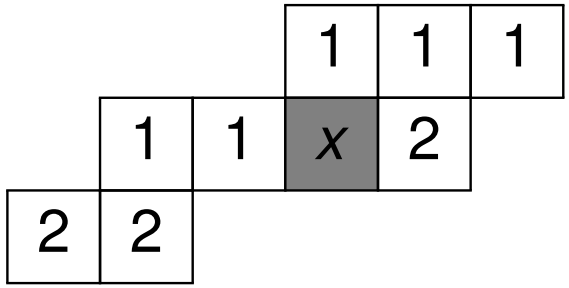
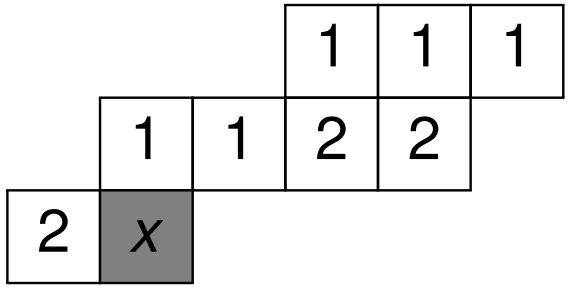


↓ one jump



Progress

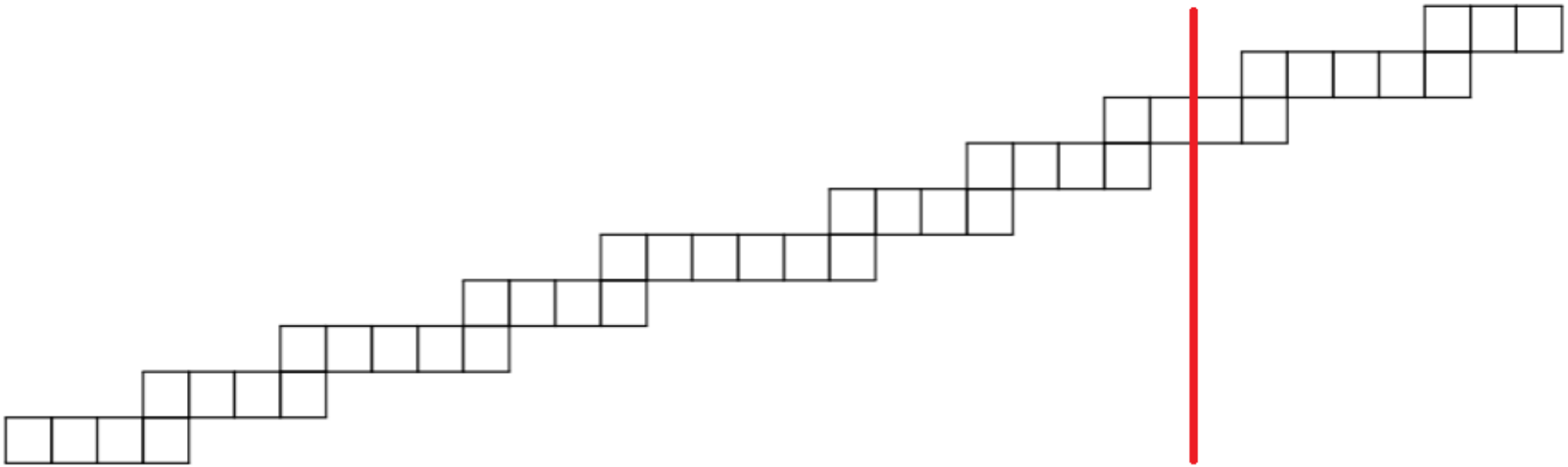
Conjecture Let T_0 be lexicographically first tableau - first tableau of a given shape, using only 1's and 2's.

Then the orbit of T has the property that

The total number of jumps = The length of orbit - 1

Progress

Method Keeping track of the **Tie line** : moves to the **left, down**



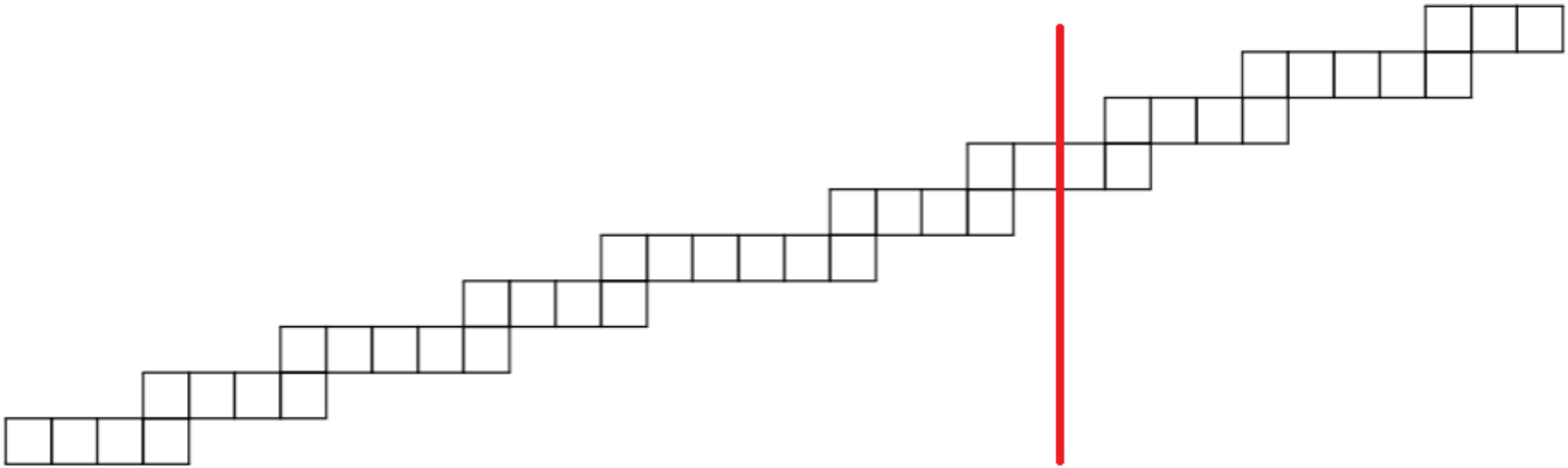
the number of 1s

=

the number of 2s

Progress

Method Keeping track of the **Tie line**



Progress

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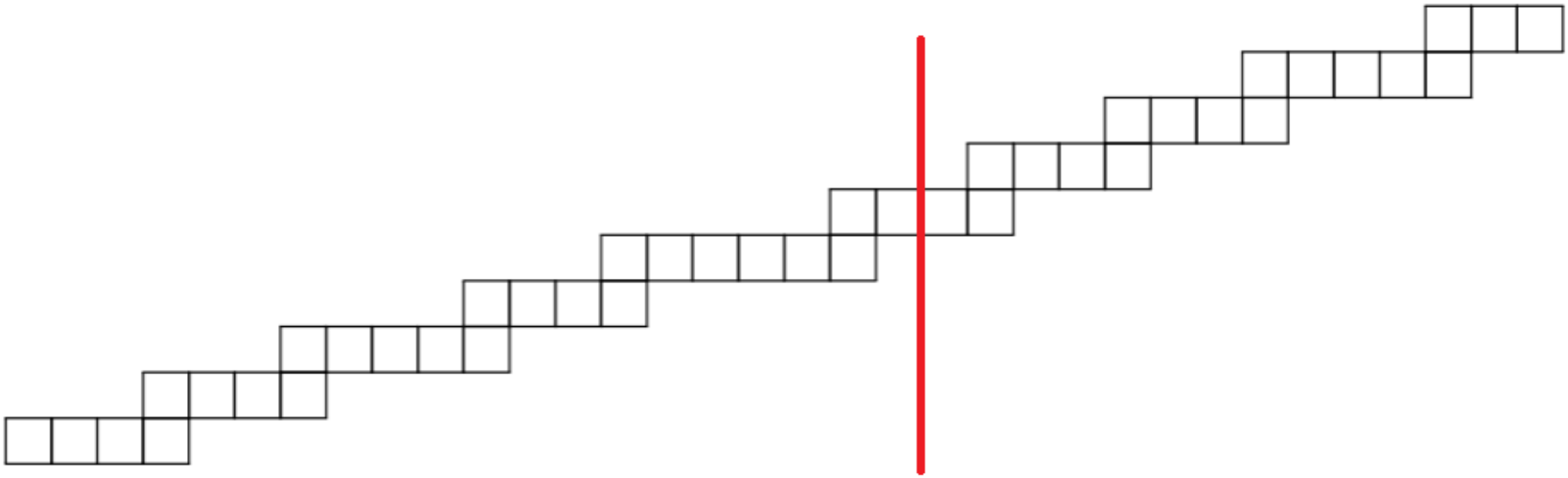

Progress

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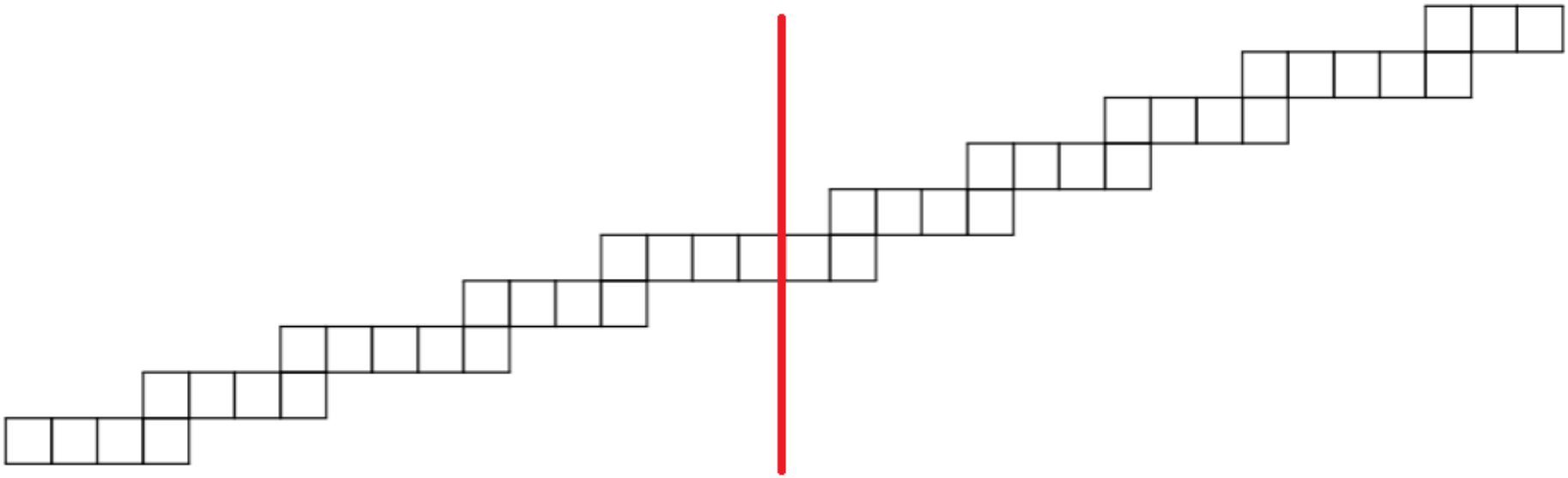

Progress

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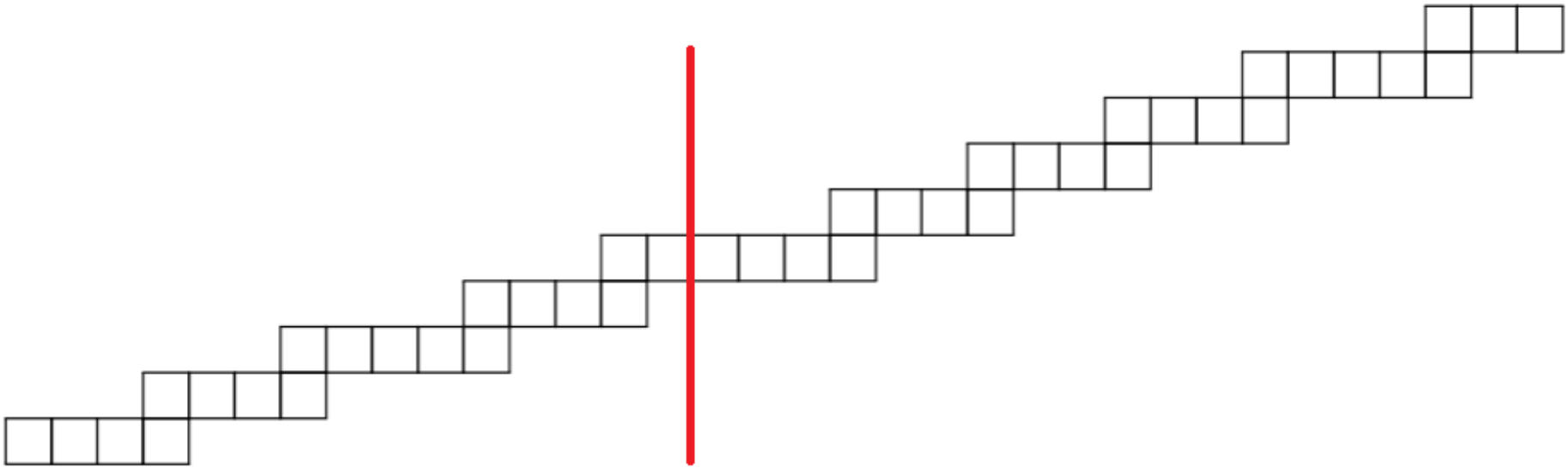


Progress

The pattern of jumps

$$[1, \ldots, 1, 0, 1, \ldots, 1, 2]$$

is repeated until the last stage of orbit when it is

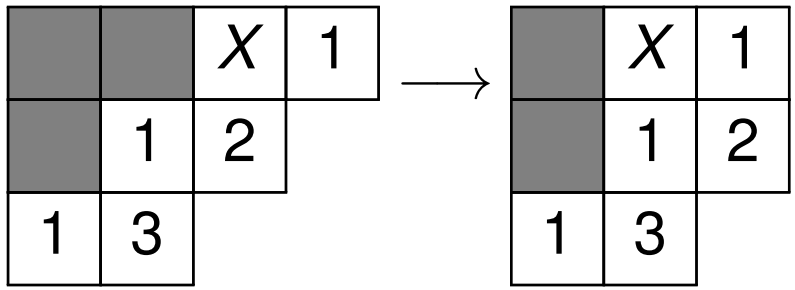
$$[1, \ldots, 1, 0, 1, \ldots, 1]$$

The total number of jumps = The length of orbit - 1

Progress

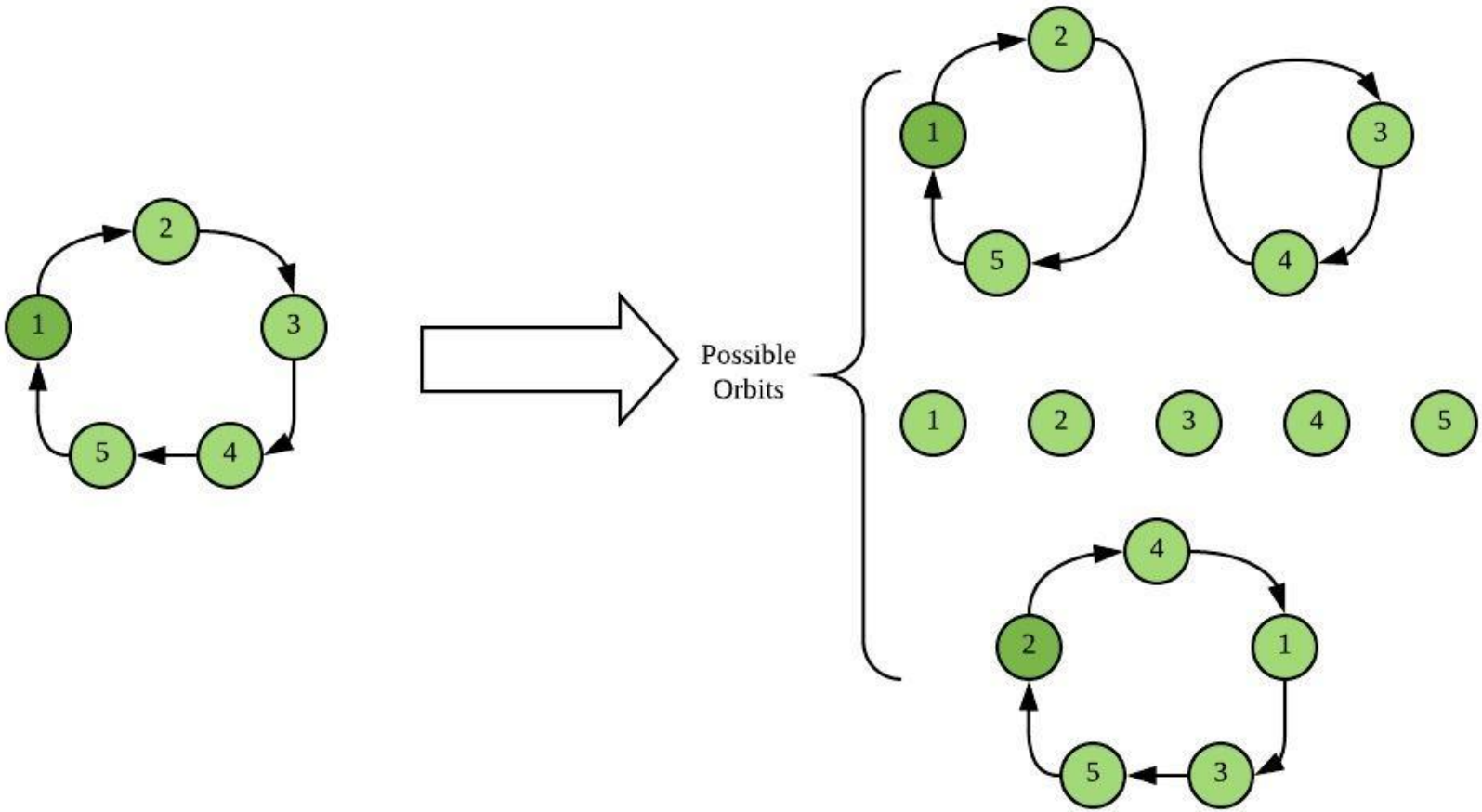
Contraction

- A small modification on tableau
- Inner shape gets changed
- Weight stays the same



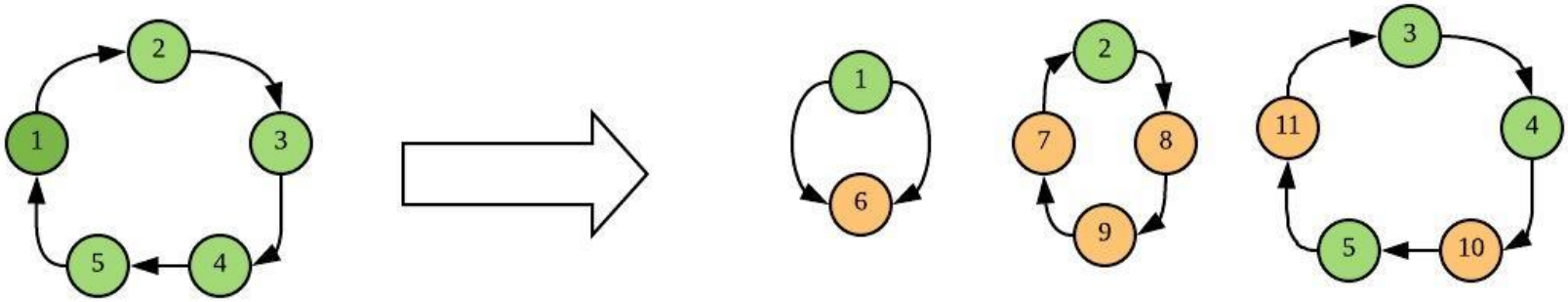
Progress

How does the orbit change if we perform contraction on orbit?
Best Case Scenario: The new orbit is organized and split into several smaller orbits



Progress

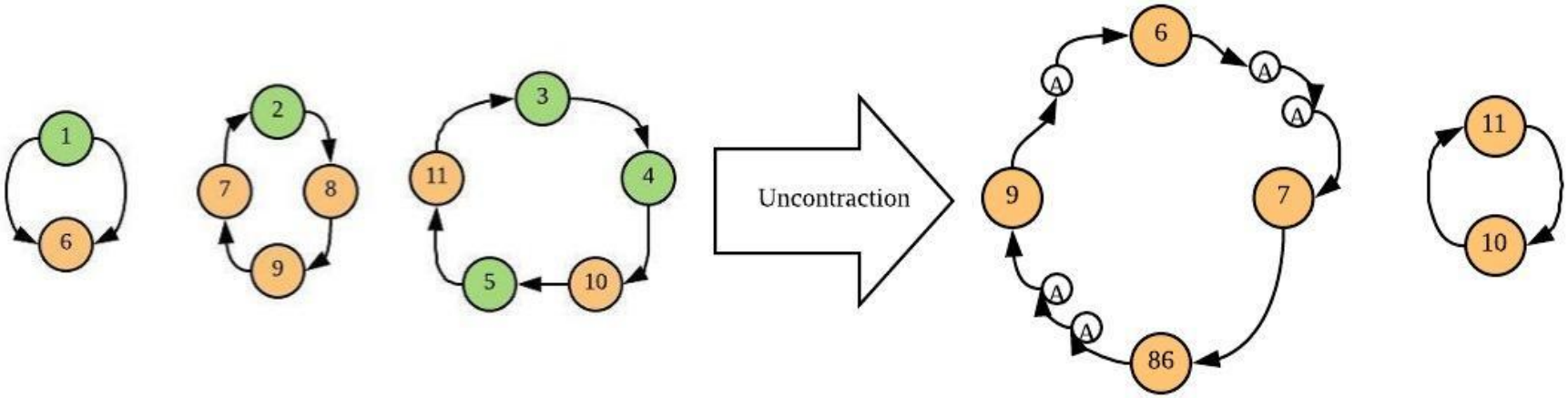
But Orbits combined in complicated ways



- In this case, we have **new** tableaux*(**in orange**)* to join in the contracted orbit
- The total length of orbit gets increased

Progress

And these added new tableaux come from other several orbits making things more complicated



- Right now, since these examples might be coincident, we need much more testings to find clear patterns