Mathematics of Gerrymandering

WXML Spring 2018

What is Gerrymandering?

“Packing & Cracking “

North Carolina’s 12th Legislative district

Image Courtesy of WPSU

Motivation: North Carolina

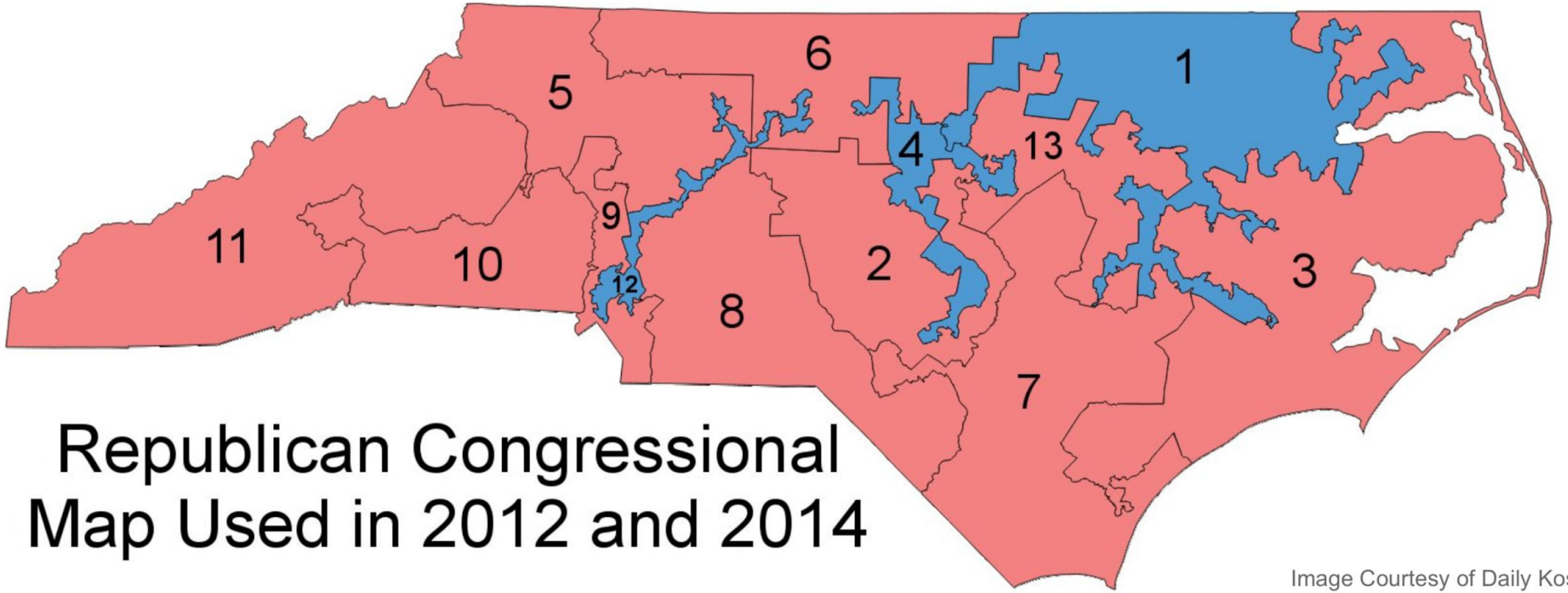


Image Courtesy of Daily Kos

Metropolis-Hastings Algorithm on Iowa

- Sampling from large space of possible Redistrictings of Iowa.
- Consider a graph which nodes are redistricting plans and edges connect redistricting only differ by 1 precinct.
- Starting with initial redistricting and random walk along graph. Determine if move on to candidate by evaluating candidate.

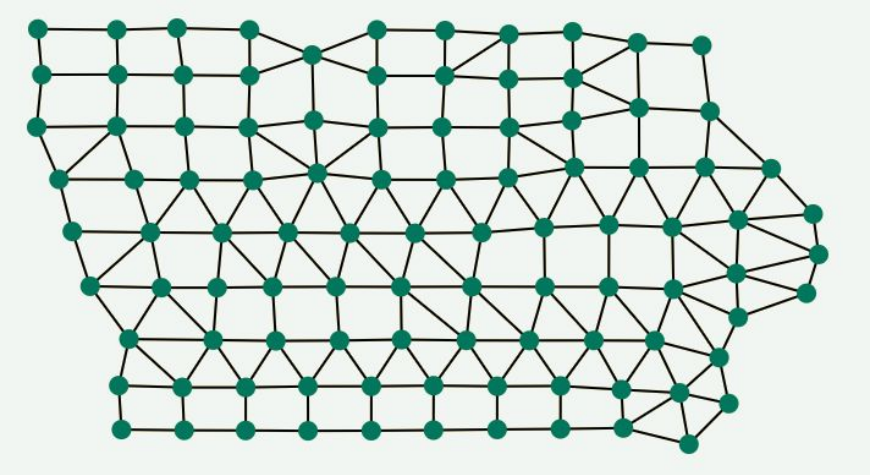


Image courtesy of Duke University.

Evaluate a candidate redistricting

States have varying types of requirements for redistricting plans. Common types include:

- **Contiguity & Compactness**
- **Even Population**
- **Minimize splits of cities and counties**
- **Voting Rights Act Compliance**

These requirements will be important for our model.

Energies - measures for evaluation

- Population energy:

$$\sum_{\text{Districts}} \left(\text{District Population} - \frac{\text{State Population}}{\text{Number of Districts}} \right)^2$$

- Compactness energy:

$$\sum_{\text{Districts}} \frac{(\text{District Perimeter})^2}{\text{District Area}}$$

We want to minimize these values.

Evaluate performance of MH algorithm

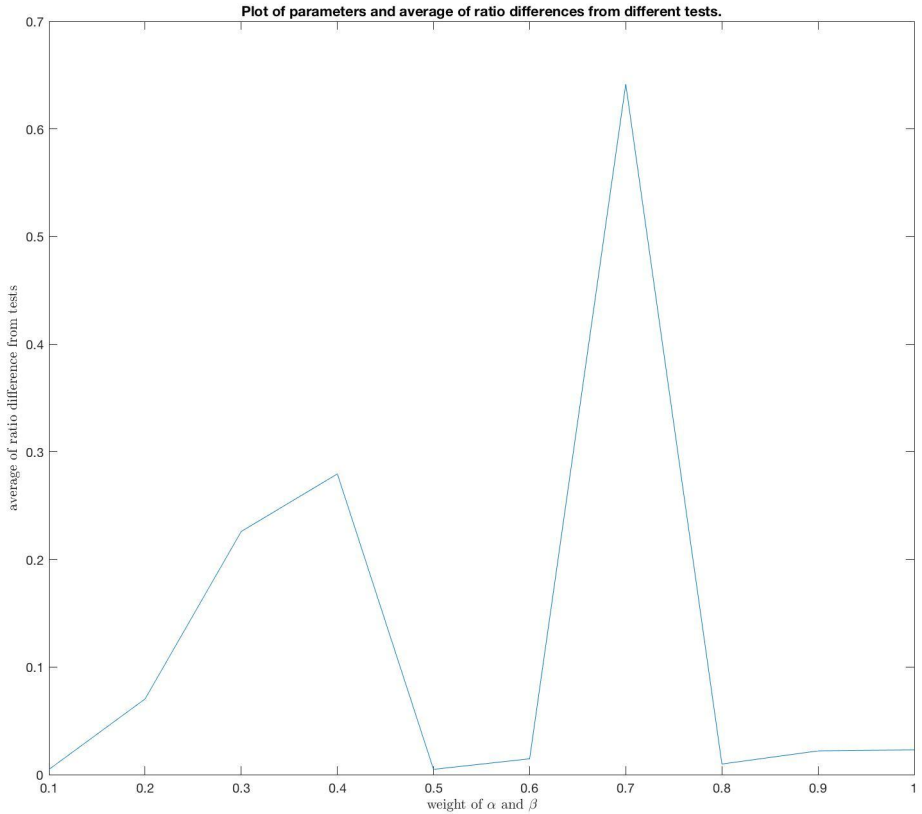
We want the samples to be independent: the final redistricting that results from the random walk should be independent of the initial map.

We are also testing different parameter values.

- Higher parameter values result in maps that more closely adhere to redistricting requirements but can drastically increase the number of iterations required for a random sample .
- Want lowest possible values that still give good samples.

We used an empirical test to determine independence.

Plot of $P(x_i=y|x_{i-1}=n)-P(x_i=y)$



Simulated Annealing

We solved this issue by incorporating simulated annealing: the process of increasing parameter values over time.

- Starting with very low parameters allows us to reach an independently generated sample quickly.
- Raising the parameters over time gives us better redistrictings. V

Timelapse of algorithm

https://github.com/weifanjiang/WXML-18wi-Research/

Future works

- Finish testing for parameters to make Iowa model work best.
- Compare new congressional district map to the actual map using past presidential election data to simulate results.
- Finish collecting data for Washington precinct map, population, and redistricting requirements.
- Apply the algorithm to Washington state.

Thank you.

Weifan Jiang
Namyoung Kim
Alex Robkin
Leo Segovia
Graduate Mentor: Tejas Devanur
Faculty Mentor: Christopher Hoffman