Rook Placement Games
Washington Experimental Mathematics Lab

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Sample game
Sample game
Sample game
Sample game

Purple wins!
The nimber of this board is 3.
In other words, it’s equivalent to a Nim pile of size 3:
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Slanted Board Theorem 1

Theorem

On a $n \times m$ slanted board, if $n \geq m$, the board will play identically to a regular rectangular board of the same dimensions. If $m$ is odd, Player 1 will win, and if $m$ is even, Player 2 will win.
Theorem

Suppose \( n \) is odd. Then Player 1 has a winning strategy if \( m \) is odd, and Player 2 has a winning strategy if \( m \) is even.
Symmetry when $m$ even, $n$ odd
Symmetry when $m$ even, $n$ odd
Symmetry when $m$ even, $n$ odd
Theorem

Suppose $B$ is a square board with the diagonal removed. Then, player 2 has a winning strategy.

- Without holes, player 1 would win.
Square Board with Diagonal Holes
Square Board with Diagonal Holes
Square Board with Diagonal Holes
Square Board with Diagonal Holes

Player 2 wins!
Future Goals

Next steps
- Complete nimber table for slanted boards
- Staircase boards
- Other boards

Challenges
- Not obvious what a good move is
- Computational Complexity
Computing the nimber

One can compute the nimber of a board $B$ recursively as follows.
Nimber(empty board) = 0.

Recursively computed nimbers of remaining board after placing rook in the given square:

```
 0 1
 0 2 2
 2 2 0
 1 0
```

Nimber(B) = Smallest nonnegative integer that does NOT appear = 3.
Nimber of remaining board = 2.