Rook Placement Games

Washington Experimental Mathematics Lab

Shruti Mokate, Angel Chen, Matt Manner
Mentors: Dr. Jonah Ostroff, Sean Griffin, Connor Ahlbach

Department of Mathematics
University of Washington

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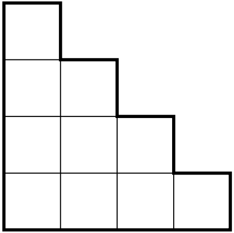
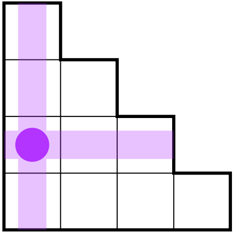
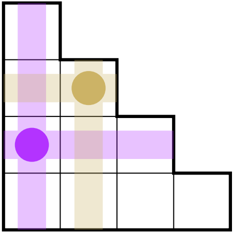
Sample game



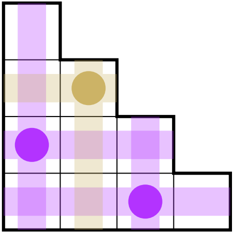
Sample game



Sample game

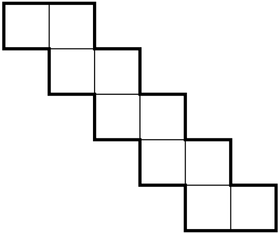


Sample game



Purple wins!

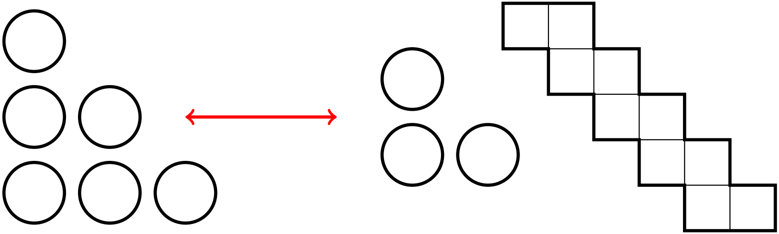
Nimbers



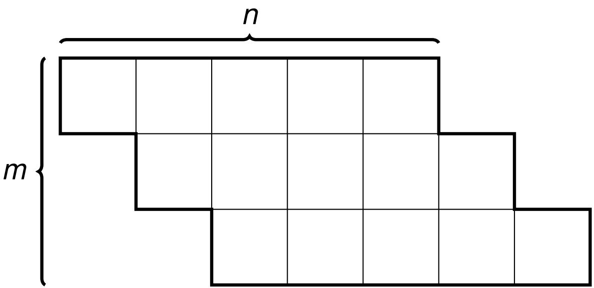
The nimber of this board is 3.

Nimbers

In other words, it's equivalent to a Nim pile of size 3:



Slanted Board



Slanted Table Results

	length of each row											
number of rows	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1
4	0	0	0	0	0	0	0	0	0	0	0	0
5	1	3	1	1	1	1	1	1	1	1	1	1
6	0	2	0	0	0	0	0	0	0	0	0	0
7	1	0	1	1	1	1	1	1	1	1	1	1
8	0	2	0		0		0	0	0	0	0	0
9	1	3	N		N		N	N	1	1	1	1
10	0	0	0		0		0		0	0	0	0
11	1	1	N		N		N		N	N	1	1
12	0	0	0		0		0		0		0	0
13	1	1	N		N		N		N		N	N
14	0	0	0		0		0		0		0	
15	1	5	N		N		N		N		N	
16	0	7	0		0		0		0		0	

Slanted Board Theorem 1

Theorem

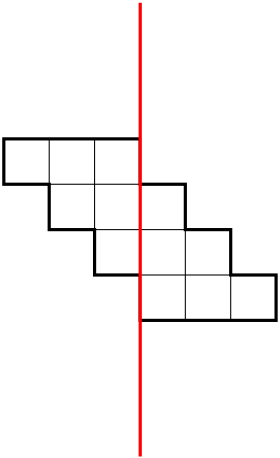
On a $n \times m$ slanted board, if $n \geq m$, the board will play identically to a regular rectangular board of the same dimensions.
If m is odd, Player 1 will win, and if m is even, Player 2 will win.

Slanted Board Theorem 2

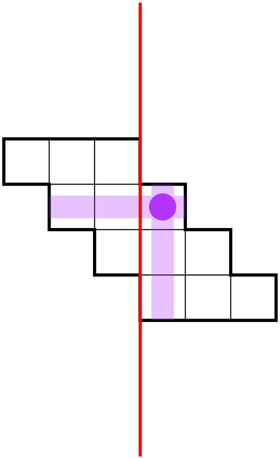
Theorem

Suppose n is odd. Then Player 1 has a winning strategy if m is odd, and Player 2 has a winning strategy if m is even.

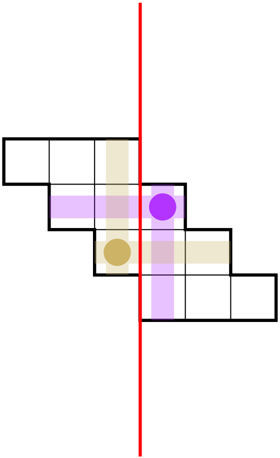
Symmetry when m even, n odd



Symmetry when m even, n odd



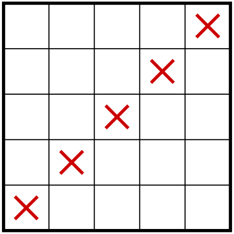
Symmetry when m even, n odd



Square Board with Diagonal Holes

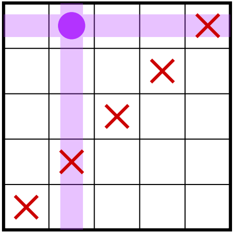
Theorem

Suppose B is a square board with the diagonal removed. Then, player 2 has a winning strategy.

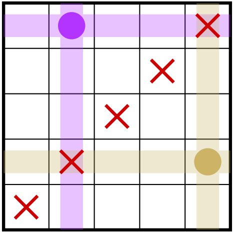


- Without holes, player 1 would win.

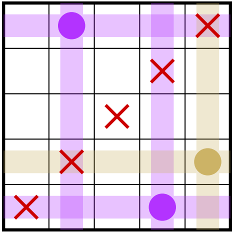
Square Board with Diagonal Holes



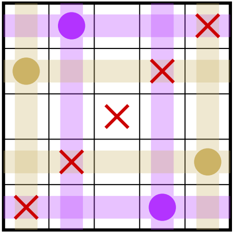
Square Board with Diagonal Holes



Square Board with Diagonal Holes



Square Board with Diagonal Holes



Player 2 wins!

Future Goals

Next steps

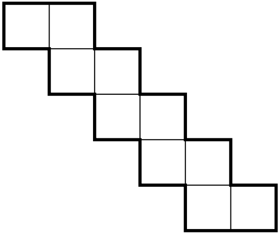
- Complete nimber table for slanted boards
- Staircase boards
- Other boards

Challenges

- Not obvious what a good move is
- Computational Complexity

Computing the nimber

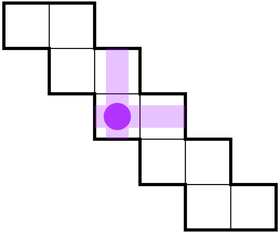
One can compute the nimber of a board B recursively as follows.



- Nimber(empty board) = 0.
- Recursively computed nimbers of remaining board after placing rook in the given square:

0	1				
	0	2			
		2	2		
			2	0	
				1	0

- Nimber(B) = Smallest nonnegative integer that does NOT appear = 3.



Nimber of remaining board = 2.