Tiling and Python Programming final report

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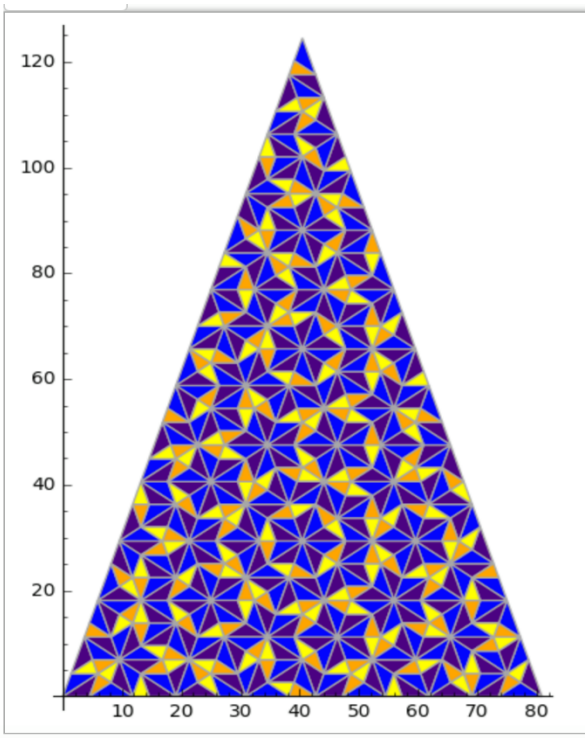
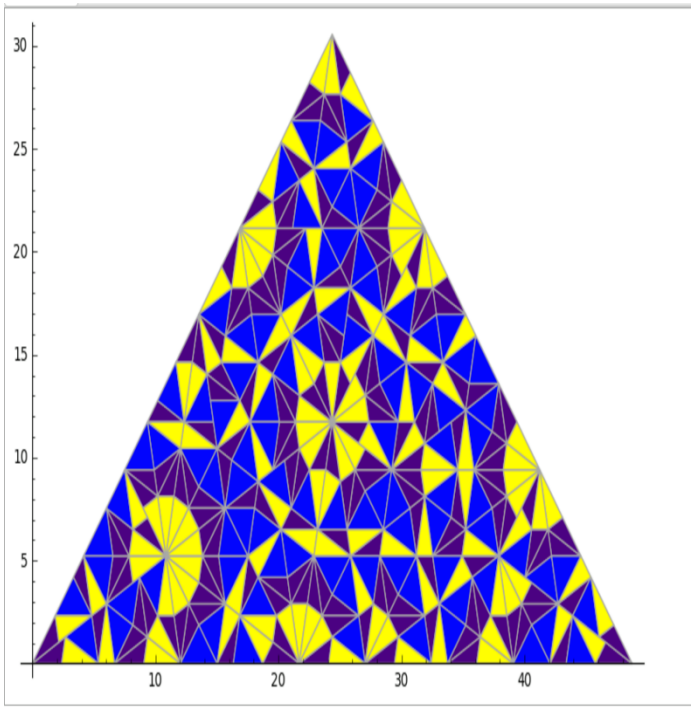
The primary goal for this research was to implement tilings in python that could then be added to a larger encyclopedia of tilings, in order to make it easy to make statistical observations of the tilings. We used the open source software CoCalc when writing and implementing the code.

To get familiar with the code we were given assignments for the first five weeks where we utilized different templates assigned by our mentors. Our task was to fill in the code for different sample tilings and understand the coding process behind each template. A substitution tiling is the process in which there is a set of tiles that become replaced with a pattern that consists of smaller versions of these set of tiles after each step in a finite plane. We identified geometric relationships within these patterns so that we could implement these tilings. The first three tiling assignments were substitution tilings named the diamond triangle tiling, the tetris tiling, and the danzer 7-fold tiling. The key geometric relationship used to complete the assignments were midpoints formulas and ratios of sides. For our last assignment, we implemented the Walton chair tiling using affine transformations. We used linear transformations to scale and rotate chair-shaped tiles that when put together, formed a larger scaled version of the chair tile.

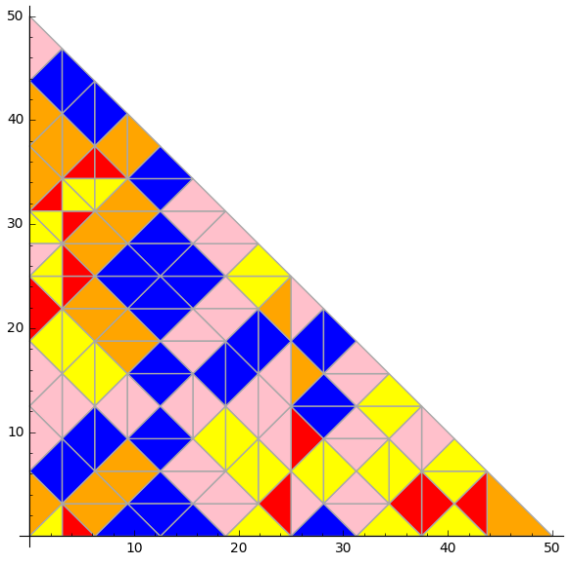
After completing these assignments, each member completed two of our own tilings. Andrew completed the Robinson Triangle tiling, which used the relationship of golden triangles (i.e. golden ratios), and the Maloney's 7 Fold tiling which used similar relationships from the

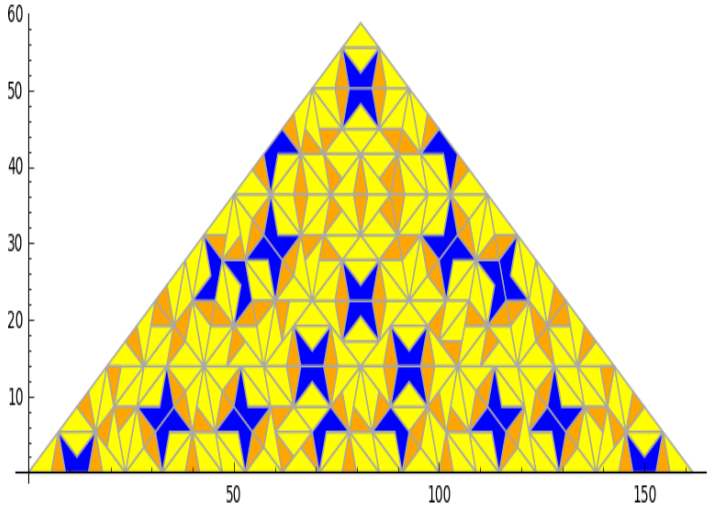
Danzer 7 Fold assignment, but required many more tiles to be accounted for in the substitution rules.



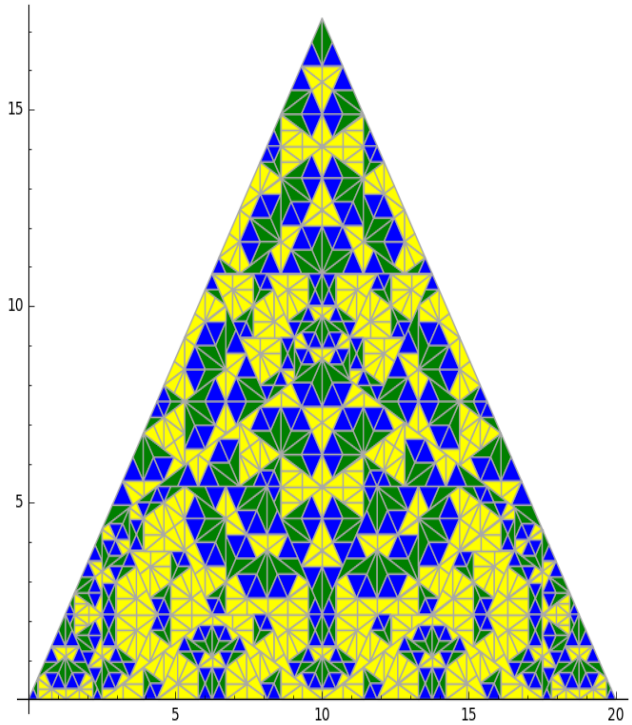


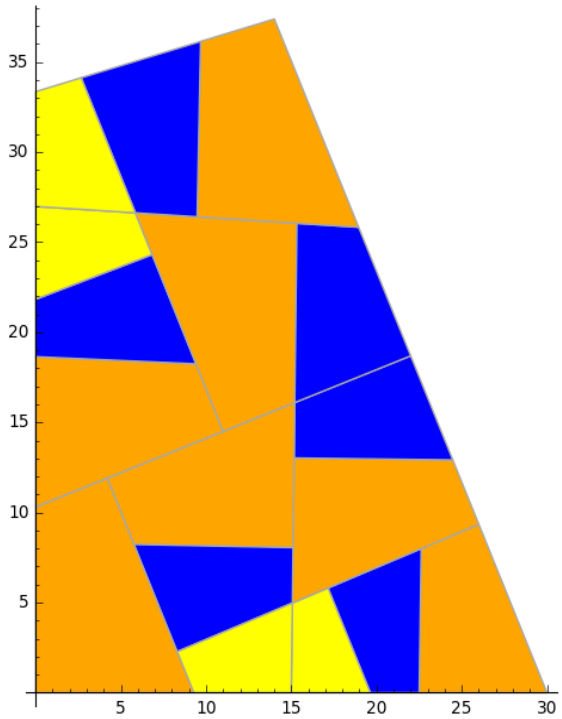
Yuqi completed the Pythia-5-1 tiling, which mainly used midpoints formulas but included more basic shapes, and the Triangle Crown tiling, which used the golden ratios between two of the three fundamental figures.





Peter completed the Chord Quadrangle tiling as well as the Sqrt-6 Triangle tiling





Along with the tilings, we investigated Ulam sets. Ulam sets originally were a sequence of natural number recursively defined where the nth term is the smallest number that can be written uniquely as the sum of two previous terms in the Ulam set. This definition can be extended to any ring equipped with a norm. We set out to investigate the patterns created by these Ulam sets when working over various rings, and seeding with various elements. For example we tried the ring R^2 with seeds of vectors that have the golden ratio as one of its components. We also tried $(Z/nZ)^2$ as a ring.

While Maxie and Kevin, our mentors, wrote and optimized most of the code to create Ulam sets, we used their code to try different seeds for the sets to get some interesting

varieties of the sets. We also used their code to present some nice visualizations of the sets to the WXML community, such as some of the following graphics