Washington Experimental Mathematics Lab

Counting K-tuples in Discrete Sets

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Background

We are interested in counting the density of integer lattice points in a ball of radius R

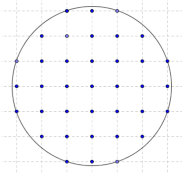

Figure: Circle with Lattice

- Count the set of primitive points (m, n) (where $gcd(m, n) = 1$)
- The density normalized for πR^2 converges to $\frac{6}{\pi^2}$

Counting Vector Pairs

- Counting pairs of integer lattice points (m_1, m_2) and (n_1, n_2) with determinant k
- Determinant is the area of the parallelogram formed by two vectors

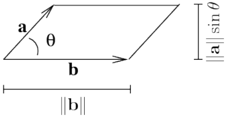

Figure: Geometric Depiction of Determinant

- Determinant:

$$\det \begin{pmatrix} m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} = m_1 n_2 - n_1 m_2 = k$$

Manipulating Determinant k

- Using $\frac{6}{\pi R^2}$ we found that for the set of unordered pairs of vectors, density is:

$$\frac{18}{\pi^2} R^4 = \sum_{k=0}^{R} c_d R^2.$$

- We want to find coefficient c_d for different determinants k

Progress

- We coded a $Count(R, K)$ function- counts the number of integer vectors within a ball (circle) of radius R that have a determinant K
- Each column contains the x and y component of a vector in the pair

Graph

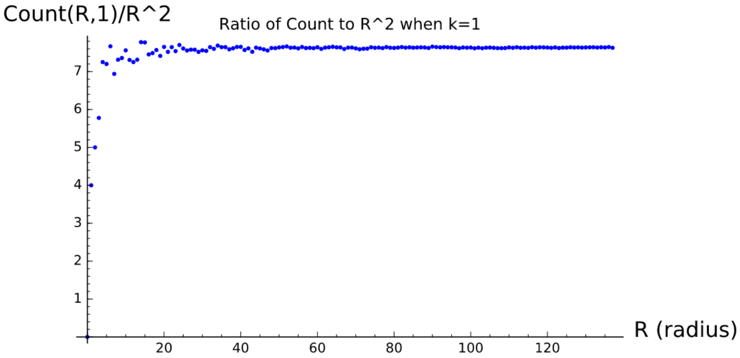

Figure: Ratio of for varying R and determinant of 1

Graphs

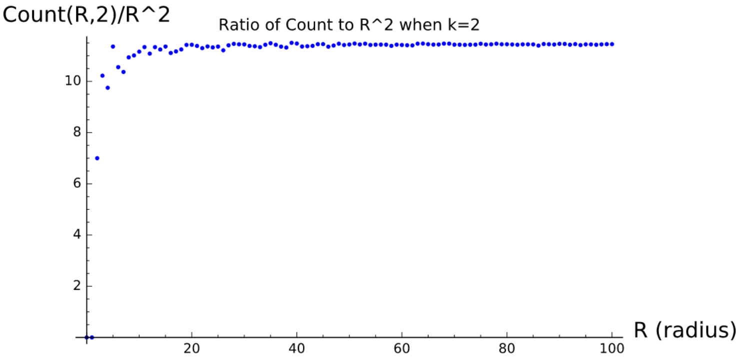

Figure: Ratio for varying R and determinant of 2

Graphs

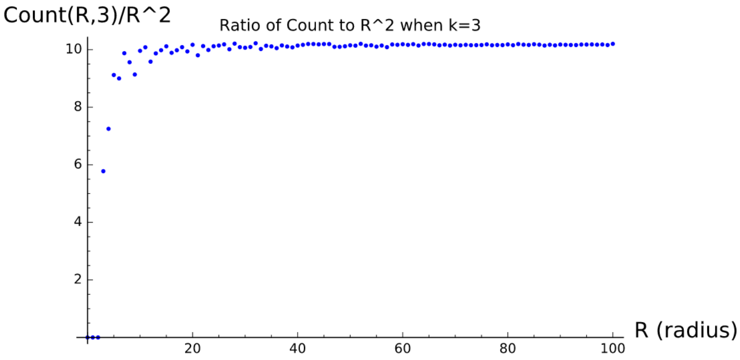

Figure: Ratio for varying R and determinant of 3

Future goals

- Discover the relationship between the Euler Phi function

$$\sum_{k \leq R} \varphi(k) \sim \frac{3}{\pi^2} R^2$$

  and our $Count(R, k)$ function as k (the determinant) changes
- Verify the number of pairs of vectors is on the order of $c_d R^2$
- Look at ratios $Count(R, k)/Count(R, k-1)$ for very large R and see what happens as k changes