Washington Experimental Mathematics Lab

Orbit Structure of Crystal Operators

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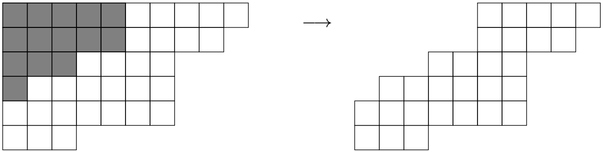
Tableau

Skew Young Diagram

Start with a large shape : [10, 9, 7, 7, 7, 3]
Remove the inner shape [5, 5, 3, 1]



Tableau

Skew Semistandard Young Tableau

Skew Diagram
Filled with positive integers
Rows weakly increasing
Columns strictly increasing

					1	1	1	1	1
					2	2	2	2	
			1	2	3	3			
	1	1	2	3	4	4			
2	3	3	3	4	5	5			
3	4	4							

Operation

Algorithm on Young tableaux: 3 phases

Phase 1: Generally the x moves **down and left.**

					1	1	1	1	1
					2	2	2	2	
		x	1	2	3	3			
	1	1	2	3	4	4			
2	3	3	3	4	5	5			
3	4	4							

Phase 1 (i=1)

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	x	2	3	4	4			
2	3	3	3	4	5	5			
3	4	4							

Phase 1 (i=2)

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
x	3	3	3	4	5	5			
3	4	4							

Phase 1 (i=3)

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
3	3	3	3	4	5	5			
x	4	4							

Phase 1 (i=4)

Operation

Algorithm on Young tableaux: 3 phases

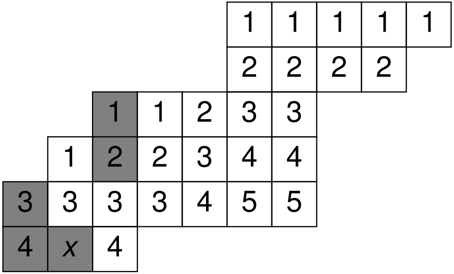
Phase 2: Generally the x moves **up and right.**

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
3	3	3	3	4	5	5			
x	4	4							

Phase 2 (i=4)

Operation

Algorithm on Young tableaux: 3 phases



Phase 2 (i=4)

Operation

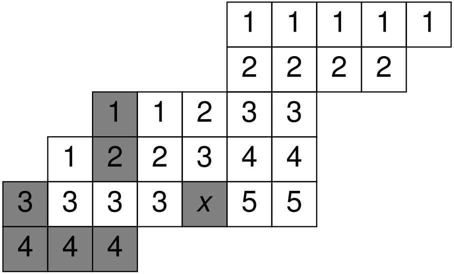
Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
3	3	3	3	4	5	5			
4	4	x							

Phase 2 (i=4)

Operation

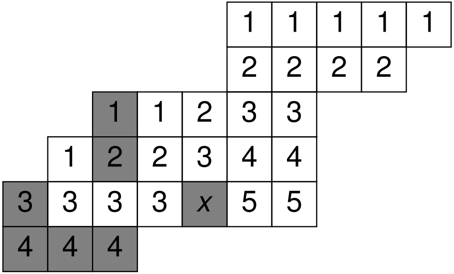
Algorithm on Young tableaux: 3 phases



Phase 2 (i=4)

Operation

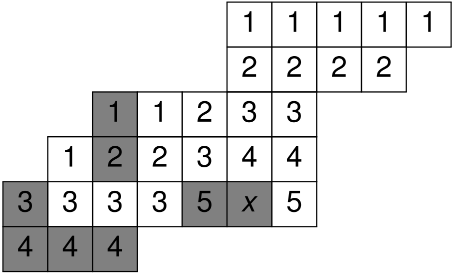
Algorithm on Young tableaux: 3 phases



Phase 2 (i=5)

Operation

Algorithm on Young tableaux: 3 phases



Phase 2 (i=5)

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
3	3	3	3	5	5	x			
4	4	4							

Phase 2 (i=5)

Operation

Algorithm on Young tableaux: 3 phases

Phase 3: Generally the x moves **up and left.**

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
3	3	3	3	5	5	x			
4	4	4							

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
3	3	3	3	5	x	5			
4	4	4							

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	3	4	4			
3	3	3	3	x	5	5			
4	4	4							

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	2	3	3			
	1	2	2	x	4	4			
3	3	3	3	3	5	5			
4	4	4							

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	1	x	3	3			
	1	2	2	2	4	4			
3	3	3	3	3	5	5			
4	4	4							

Operation

Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		1	x	1	3	3			
	1	2	2	2	4	4			
3	3	3	3	3	5	5			
4	4	4							

Operation

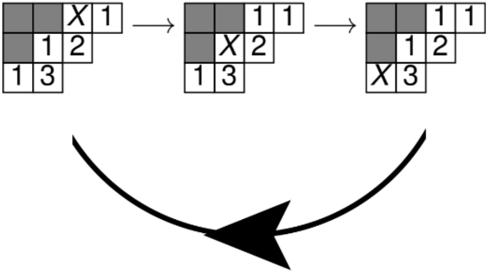
Algorithm on Young tableaux: 3 phases

					1	1	1	1	1
					2	2	2	2	
		x	1	1	3	3			
	1	2	2	2	4	4			
3	3	3	3	3	5	5			
4	4	4							

Recap

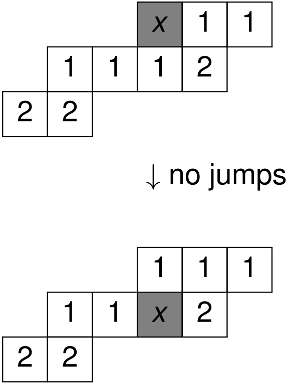
Orbit :

These three phases create a **new tableau.** After several iterations, we generate a complete orbit:



Progress 1

What is a Jump?



Progress 1

What is a Jump?

			1	1	1
	1	1	x	2	
2	2				

$\downarrow$ one jump

			1	1	1
	1	1	2	2	
2	x				

Progress 1

Conjecture (from Geometry)
the total number of jumps in orbit $\geq$ the length of orbit - 1

Question When does equality hold?

Progress 1

Focusing on tableaux containing only 1's and 2's

Let T_0 be **lexicographically** first tableau
$\rightarrow$ all 1's and x are located **as high as possible**

Hypothesis The following equality holds for T_0 's orbit

The total number of jumps = The number of tableaux - 1

Progress

Contraction

- A way to make **small change** to the tableau
- Start with an inner square, do "Inverse JDT" to slide the square to the outer edge of the tableau

$$T = \begin{array}{|c|c|c|c|} \hline \cellcolor{gray} & \cellcolor{gray} & X & 1 \\ \hline \cellcolor{gray} & 1 & 2 \\ \cline{1-3} 1 & 3 \\ \cline{1-2} \end{array} \longrightarrow T' = \begin{array}{|c|c|c|} \hline \cellcolor{gray} & X & 1 \\ \hline \cellcolor{gray} & 1 & 2 \\ \hline 1 & 3 \\ \cline{1-2} \end{array}$$

How does orbit of T compare to orbit of T'?