Rook Placement Games

Washington Experimental Mathematics Lab

Shruti Mokate, Angel Chen, Matt Manner
Mentors: Dr. Jonah Ostroff, Sean Griffin, Connor Ahlbach

Department of Mathematics
University of Washington

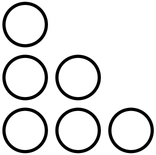
April 26, 2018

Impartial Games

- 2 players with same rules
- Finite steps
- No random chance
- No secrets
- Whoever goes last wins

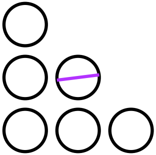
Example - Nim

- Most well-known impartial game
- Example



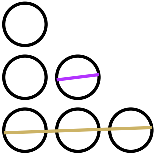
Example - Nim

- Most well-known impartial game
- Example



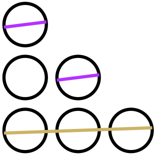
Example - Nim

- Most well-known impartial game
- Example



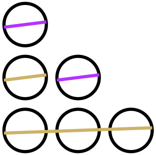
Example - Nim

- Most well-known impartial game
- Example



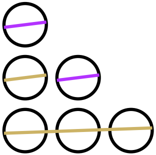
Example - Nim

- Most well-known impartial game
- Example



Example - Nim

- Most well-known impartial game
- Example



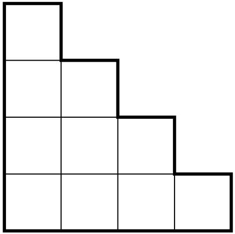
- Grundy-Sprague theorem: every game with the qualities on the previous page is equivalent to some game of Nim.
- The **nimber** of a game tells you what it's equivalent to.

Rook Placement Game

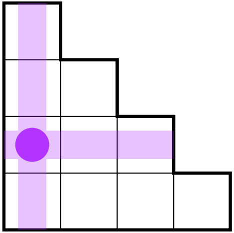
- Start with a board B, a finite collection of cells on a grid.
- Two players take turns placing rooks on B so that no two rooks *attack* each other.
- Whoever places the last rook wins.

Question: Given the board B, who wins? What's the nimber?

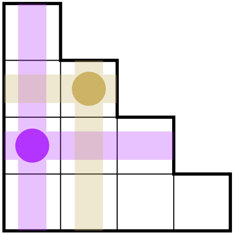
Example game



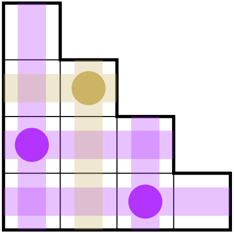
Example game



Example game



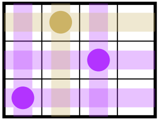
Example game



Purple wins!

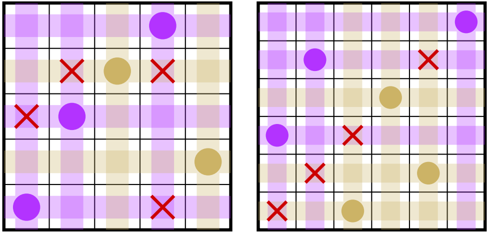
Rectangular Boards

- Rectangular boards are boring: If B is an $m \times n$ board with $m \leq n$, then player 1 wins if m is odd, and player 2 wins if m is even.



- What if there are **holes** in the board?

Results



Theorem

Let B be an $m \times n$ rectangular board with $m \leq n$, and let B' be a board obtained from B by removing at most $n-2$ cells if m is even, and $n-1$ cells if m is odd. Then B and B' will have the same winner.

Future Goals

Next steps

- Find all placements of the minimum number of holes to change the winner for rectangular boards.
- Staircase boards
- Other boards

Challenges

- Not obvious what a good move is
- Computational Complexity