DQG: Dual Quaternion Guidance for the SPLICE Project

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Overview

The SPLICE Project

Problem Formulation

Sequential Convex Programming Implementation

Future Additions and Improvements



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Safe & Precise Landing - Integrated Capabilities Evolution

DQG: SPLICE Guidance

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DQG Algorithm Development

University of Washington:

- Basic research on:
 - solution of nonconvex OCPs
 - problem formulations for lunar descent
- Develop Matlab-based toolbox to test and compare formulations and solution methods
- Develop flight code prototype in C/C++

Team:

TPR, M. Szmuk, D. Malyuta M. Mesbahi, B. Açıkmeşe

Draper Laboratory:

- Develop cFS application for DQG
- Implement DQG prototype on SPLICE compute hardware

Team:

Javier Doll

Matt Fritz, Tim Barrows





DQG: SPLICE Guidance

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Dual Quaternions

- A unit dual quaternion, \tilde{q} , satisfies two properties:

$$\tilde{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}_{8 \times 1} \quad \text{where} \quad q_1^\top q_1 = 1, \quad \text{and} \quad q_1^\top q_2 = 0.$$

- DQG uses a right-handed, Hamiltonian, scalar-last convention
- The geometry of unit dual quaternions in lower dimensions:





Dual Quaternions & Rigid Body Motion

- The unit dual quaternion \tilde{q} represents the pose of $\mathcal{F}_{\mathcal{B}}$ with respect to $\mathcal{F}_{\mathcal{I}}$:

$$\tilde{q} = \begin{bmatrix} q \\ \frac{1}{2}q \otimes r_{\mathcal{B}} \end{bmatrix} \equiv \begin{bmatrix} q \\ \frac{1}{2}r_{\mathcal{I}} \otimes q \end{bmatrix} \quad \text{and} \quad \tilde{\omega} = \begin{bmatrix} \omega_{\mathcal{B}} \\ v_{\mathcal{B}} \end{bmatrix}$$

- The dual velocity $\tilde{\omega}$ is composed of angular velocity and $v_{\mathcal{B}} = \dot{r}_{\mathcal{B}} + \omega_{\mathcal{B}}^{\times} r_{\mathcal{B}}$



Cost Function

 $\max \quad m(t_f)$

Dual Quaternion Dynamics

$$\begin{split} \dot{m} &= -\alpha \| u_{\mathcal{B}} \|_2 \qquad [\text{NC}] \\ \dot{\tilde{q}} &= \frac{1}{2} \tilde{q} \otimes \tilde{\omega} \qquad [\text{NC}] \\ J \dot{\tilde{\omega}} &= \Phi u_{\mathcal{B}} - \tilde{\omega} \oslash J \tilde{\omega} + m \tilde{g}_{\mathcal{B}} \qquad [\text{NC}] \end{split}$$

[C]

- 15-dimensional state vector (1+8+6)
- $u_{\mathcal{B}}$ is thrust vector, $g_{\mathcal{B}}$ is gravity
- matrix Φ maps thrust to a force & torque



Cost Function max $m(t_f)$ **Dual Quaternion Dynamics** $\dot{m} = -\alpha \|u_{\mathcal{B}}\|_2$ [NC] $\dot{\tilde{q}} = \frac{1}{2}\tilde{q}\otimes\tilde{\omega}$ [NC] $J\dot{\tilde{\omega}} = \Phi u_{\mathcal{B}} - \tilde{\omega} \oslash J\tilde{\omega} + m\tilde{a}_{\mathcal{B}}$ [NC] **Control Constraints** $u_{\min} < \|u_{\mathcal{B}}\|_2 < u_{\max}$ [NC] $||u_{\mathcal{B}}||_{2} \leq \sec \delta_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}}$ [C] $-\dot{u}_{z,\max} \leq z_{\mathcal{B}}^{\top} u_{\mathcal{B}} \leq \dot{u}_{z,\max}$ [C] $||E_{xy}u_{\mathcal{B}}||_2 < \dot{\delta}_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}}$ [C]

[C]

- upper/lower throttle constraint
- gimbal angle constraint, $z_{\mathcal{B}}$ is vertical direction
- approx. throttle rate constraint
- approx. gimbal angle constraint



Cost Function

max $m(t_f)$

[C]

Dual Quaternion Dynamics

$\dot{m} = -\alpha \ u_{\mathcal{B}}\ _2$	[NC]
$\dot{ ilde{q}}=rac{1}{2} ilde{q}\otimes ilde{\omega}$	[NC]
$J\dot{\tilde{\omega}} = \Phi u_{\mathcal{B}} - \tilde{\omega} \oslash J\tilde{\omega} + m\tilde{g}_{\mathcal{B}}$	[NC]

Control Constraints

$$\begin{aligned} u_{\min} &\leq \|u_{\mathcal{B}}\|_{2} \leq u_{\max} & [\mathsf{NC}] \\ \|u_{\mathcal{B}}\|_{2} &\leq \sec \delta_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}} & [\mathsf{C}] \\ -\dot{u}_{z,\max} &\leq z_{\mathcal{B}}^{\top} u_{\mathcal{B}} \leq \dot{u}_{z,\max} & [\mathsf{C}] \\ \|E_{xy} u_{\mathcal{B}}\|_{2} &\leq \dot{\delta}_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}} & [\mathsf{C}] \end{aligned}$$

State Constraints

$$\begin{split} -\tilde{q}^{\top} M_{\gamma} \tilde{q} + \|2E_{d} \tilde{q}\|_{2} \cos \gamma_{\max} &\leq 0 \qquad [\mathsf{C}] \\ \tilde{q}^{\top} M_{\theta} \tilde{q} + \cos \theta_{\max} &\leq 0 \qquad [\mathsf{C}] \\ m_{dry} &\leq m \qquad [\mathsf{C}] \\ \|E_{v} \tilde{\omega}\|_{2} &\leq v_{\max} \qquad [\mathsf{C}] \\ \|E_{w} \tilde{\omega}\|_{\infty} &\leq \omega_{\max} \qquad [\mathsf{C}] \end{split}$$

- approach angle (glide slope) and tilt angle expressed as quadratic functions of \tilde{q}

- mass, speed, and angular rate constraints



Cost Function

$m(t_f)$ \max

Dual Quaternion Dynamics

$\dot{m} = -\alpha \ u_{\mathcal{B}}\ _2$	[NC]
$\dot{ ilde{q}} = rac{1}{2} \widetilde{q} \otimes \widetilde{\omega}$	[NC]
$J\dot{\tilde{\omega}} = \Phi u_{\mathcal{B}} - \tilde{\omega} \oslash J\tilde{\omega} + m\tilde{g}_{\mathcal{B}}$	[NC]

Control Constraints

$$\begin{aligned} u_{\min} &\leq \|u_{\mathcal{B}}\|_{2} \leq u_{\max} & [\mathsf{NC}] \\ \|u_{\mathcal{B}}\|_{2} &\leq \sec \delta_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}} & [\mathsf{C}] \\ -\dot{u}_{z,\max} &\leq z_{\mathcal{B}}^{\top} u_{\mathcal{B}} \leq \dot{u}_{z,\max} & [\mathsf{C}] \\ E_{xy} u_{\mathcal{B}}\|_{2} &\leq \dot{\delta}_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}} & [\mathsf{C}] \end{aligned}$$

State Constraints

$-\tilde{q}^{\top} M_{\gamma} \tilde{q} + \ 2E_d \tilde{q}\ _2 \cos \gamma_{\max} \le 0$	[C]
$\tilde{q}^{\top} M_{\theta} \tilde{q} + \cos \theta_{\max} \le 0$	[C]
$m_{dry} \leq m$	[C]
$\ E_v \tilde{\omega}\ _2 \le v_{\max}$	[C]
$\ E_w \tilde{\omega}\ _{\infty} \le \omega_{\max}$	[C]

Boundary Conditions		
$m(t_0)=m_{\it ic}$		[C]
$\tilde{q}(t_0) = b_{\tilde{q}}(q(t_0)),$	$b_f(\tilde{q}(t_f)) \le 0$	[C]
$\tilde{\omega}(t_0) = b_{\tilde{\omega}}(q(t_0)),$	$\tilde{\omega}(t_f) = \tilde{\omega}_f$	[NC]



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[C] [C]

[C]



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Sequential Convex Programming



- Solve nonconvex OCP by solving a sequence of convex approximations
- ▶ Initial guess obtained by simply interpolating between current and desired final states
- ► Converged solution satisfies dynamics to pre-determined precision



M. Szmuk et al. JGCD, 2020 | TPR et al. JGCD, 2020.

Solve Step

DQG: SPLICE Guidance

- Each iteration solves a second-order cone program using custom solver BSOCP¹
- BSOCP is written in C++, but can generate C code tailored to the specific problem

OCP

- convex cost function
- nonlinear, differential eq. constraints
- general ineq. and eq. constraints

$$\begin{array}{ll} \min_{u,p} & J(x,u,p) \\ \text{s.t.} & \dot{x} = f(x,u,p) \\ & 0 \geq g(x,u,p) \\ & 0 = h(x,u,p) \end{array}$$

¹D. Dueri, 2018

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SOCP

- linear cost function
- affine, algebraic eq. constraints
- affine or second-order cone ineq. constraints

$$\min_{z} \quad c^{\top} z$$
s.t. $Az = b$
 $z \in \mathcal{C}_L \times \mathcal{C}_{Q_1} \times \cdots \times \mathcal{C}_{Q_m}$

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Solve Step

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OCP

- convex cost function
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SOCP

Convexification

- linear cost function
- affine, algebraic eq. constraints
- affine or second-order cone ineq. constraints

 $\min_{u,p} \quad J(x, u, p) \qquad \qquad \min_{z} \quad c^{\top}z \\ \text{s.t.} \quad \dot{x} = f(x, u, p) \qquad \qquad \text{s.t.} \quad Az = b \\ 0 \ge g(x, u, p) \qquad \qquad z \in \mathcal{C}_L \times \mathcal{C}_{Q_1} \times \cdots \times \mathcal{C}_Q \\ 0 = h(x, u, p) \qquad \qquad z \in \mathcal{C}_L \times \mathcal{C}_{Q_1} \times \cdots \times \mathcal{C}_Q$

¹D. Dueri, 2018

T. P. Reynolds



Solve Step

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OCP

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$$\min_{u,p} \quad J(x, u, p)$$
s.t. $\dot{x} = f(x, u, p)$

$$0 \ge g(x, u, p)$$

$$0 = h(x, u, p)$$

¹D. Dueri, 2018

SOCP

- linear cost function
- affine, algebraic eq. constraints
- affine or second-order cone ineq. constraints

$$\begin{array}{ll} \min_{z} & c^{\top}z \\ \text{s.t.} & Az = b \\ & z \in \mathcal{C}_{L} \times \mathcal{C}_{Q_{1}} \times \cdots \times \mathcal{C}_{Q_{m}} \end{array}$$

Parsing



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Convexification: Propagation

- Assume the control can be affinely interpolated between a set of \boldsymbol{N} time nodes
- Use an exact discretization; solution of linearized ODE via numerical integration
- Defects Δ serve as an indicator of dynamic feasibility
- Properly implemented, this should take roughly $\mathcal{O}(1)\%$ of the DQG's runtime



Convexification

step

Convexification: Constraint Approximation

Control Constraints

$\begin{aligned} u_{\min} &\leq \|u_{\mathcal{B}}\|_{2} \leq u_{\max} \\ \|u_{\mathcal{B}}\|_{2} \leq \sec \delta_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}} \\ -\dot{u}_{z,\max} \leq z_{\mathcal{B}}^{\top} u_{\mathcal{B}} \leq \dot{u}_{z,\max} \\ \|E_{xy} u_{\mathcal{B}}\|_{2} \leq \dot{\delta}_{\max} z_{\mathcal{B}}^{\top} u_{\mathcal{B}} \end{aligned}$

State Constraints

$$\begin{split} -\tilde{q}^{\top} M_{\gamma} \tilde{q} + \|2E_{d} \tilde{q}\|_{2} \cos \gamma_{\max} &\leq 0 \qquad [\mathsf{C}] \quad (\textit{linearized}) \\ \tilde{q}^{\top} M_{\theta} \tilde{q} + \cos \theta_{\max} &\leq 0 \qquad [\mathsf{C}] \quad (\textit{SOC}) \\ m_{\textit{dry}} &\leq m \qquad [\mathsf{C}] \quad (\textit{affine}) \\ \|E_{v} \tilde{\omega}\|_{2} &\leq v_{\max} \qquad [\mathsf{C}] \quad (\textit{SOC}) \\ \|E_{w} \tilde{\omega}\|_{\infty} &\leq \omega_{\max} \qquad [\mathsf{C}] \quad (\textit{affine}) \end{split}$$

[NC] (linearized + SOC)

[] *(SOC)*

[C] *(affine)* [C] *(SOC)*

- Approach cone constraint is not an SOC, must linearize
- Max. violations of approx'd constraints tracked using parameter: δ
- Each constraint enforced at \boldsymbol{N} nodes
- To parse SOCP: can quantify the min. number of variables and rows of $A,\,b$



Convexification: Constraint Approximation



- Specified:
 - initial mass, (inertial) position and velocity, angular rates
 - final attitude, (inertial) velocity, angular rates

$$b_{\tilde{q}}(q(t_0)) = \begin{bmatrix} q(t_0) \\ \frac{1}{2}r_{\mathcal{I}} \otimes q(t_0) \end{bmatrix}$$
$$b_{\tilde{\omega}}(q(t_0)) = \begin{bmatrix} \omega_{\mathcal{B}}(t_0) \\ q(t_0)^* \otimes v_{\mathcal{I}}(t_0) \otimes q(t_0) \end{bmatrix}$$
$$b_f(\tilde{q}(t_f)) = \|2E_d\tilde{q}(t_f) - c'\|_2 - \varepsilon_{\text{miss}}$$
where $E_d = \text{diag} \{0_{4\times 4}, I_4\}$

Convexification

step

Trust Regions and Virtual Control

- Algorithmic modifications to ensure:
 - Each SOCP is feasible and bounded
 - Iterates kept "close" to the reference used for approximation
- Virtual control added to all approximated constraints:

 $[\mathsf{NC}]: \quad h(x) = 0 \qquad \Longrightarrow \qquad [\mathsf{C}]: \quad h(\bar{x}) + \nabla h(\bar{x})^{\top} (x - \bar{x}) + \nu = 0$

where ν is *unconstrained* but highly *penalized* in the cost.

- Trust region added as additional constraints:

$$\|x - \bar{x}\|_2^2 + \|u - \bar{u}\|_2^2 \le \eta$$
 and $\|p - \bar{p}\|_2^2 \le \eta_p$

where η,η_p are chosen by the solver and modestly penalized in the cost.

Sost Function
$$\max \quad m(t_f) \quad - \quad w_{tr}^\top \eta \quad - \quad w_{tr,p} \eta_p \quad - \quad w_{vc} \sum_k \|\nu_k\|_1$$

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Solve

step



is unbounded without a trust region

tion with approximated constraint \bar{h}

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Stopping Criteria

Stopping criteria

Criteria:

	1	small defects & constraint violation	$\max\left\{\max_{k} \Delta_{k}, \delta\right\} \leq \varepsilon_{\Delta}$
	2	small state change	$\max_k \ x_k - \bar{x}_k\ _2 \le \varepsilon_x$
	3	small final mass change	$ m_N - \bar{m}_N \le \varepsilon_m$
Logic:		1 AND (2 OF	R 3)

- $\blacktriangleright\ \varepsilon_\Delta$ measures "feasibility", both dynamic and approx'd constraints
- $\blacktriangleright\ \varepsilon_m$ used to stop if optimality not sufficiently improving
- \blacktriangleright Control changes ignored: large thrust change can have small impact on state trajectory



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Mission-Specific Initial Guess

- Currently, DQG uses a very simple initialization:

$$x_k = \left(\frac{N-k}{N-1}\right) x_{ic} + \left(\frac{k-1}{N-1}\right) x_{fc} \text{ and } u_k = m_k g_{\mathcal{B}}, \quad k = 1, \dots, N$$

where x_{ic} and x_{fc} are boundary conditions, $g_{\mathcal{B}}$ is gravity in body frame $\mathcal{F}_{\mathcal{B}}$

- This trajectory is not feasible but works well enough and is easy to compute

Better initial guess \Rightarrow fewer DQG iterations \Rightarrow lower runtime

- Idea: For a given mission, create a map from initial condition to initial guess:

$$\Psi: \mathcal{D} \to \mathcal{X} \times \mathcal{U} \times \mathbb{R}_{++}$$
$$x_0 \mapsto \{x_k, u_k, t_f\}_{k=1}^N$$

DQG: SPLICE Guidance

 \mathcal{D}

Custom Conversion to Standard Form

- BSOCP solves a problem in standard form:

$$\begin{array}{ll} \min & c^{\top}z \\ \text{s.t.} & Az = b \\ & z \in \mathcal{C}_L \times \mathcal{C}_{Q_1} \times \cdots \times \mathcal{C}_{Q_m} \end{array}$$

- Two possible methods to define data A, b, c and the cone dimensions:

Generic Parser (Current)

- Ideal for prototyping
- X Can add variables/constraints
- X Opaque problem construction
- X Uses dynamic memory allocation

Handparsing

- X Least flexible
- ✓ Guarantees smallest problem
- Most control over coding
- Uses static memory allocation



Handparsing to Standard Form



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Handparsing to Standard Form



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State-Triggered Constraints (STC)

- STCs are constraints enforced conditionally based on the value of a trigger function



- Equivalently, we can enforce the nonconvex constraint

 $-\min(g(z), 0) h(z) \le 0$

- Models binary decisions using continuous variables
- Can combine trigger/constraint conditions using Boolean AND and OR operations

M. Szmuk et al. JGCD, 2020

State-Triggered Constraints (STC)

- Reconciles vehicle configuration with feasibility of optimal control problem
- Trigger: slant range larger than ρ

 $g(\tilde{q}) = \rho - \|2E_d\tilde{q}\|_2$

- Constraint: line of sight angle to landing target

 $h(\tilde{q}) = \tilde{q}^{\top} M_{\xi} \tilde{q} + \|2E_d \tilde{q}\|_2 \cos \xi_{\max} - \varepsilon$



State-Triggered Constraints (STC)

- Lunar descent orbits characteristically low altitude at large downrange distances
- Trigger: slant range small than ρ

 $q(\tilde{q}) = \|2E_d\tilde{q}\|_2 - \rho$

- Constraint: approach angle to landing target

 $h(\tilde{q}) = -\tilde{q}^{\top} M_{\gamma} \tilde{q} + \|2E_d \tilde{q}\|_2 \cos \gamma_{\max}$



DQG: SPLICE Guidance

Summary

➡ DQG solves a nonconvex optimal control problem in real-time

- parameterizes pose with dual quaternions
- includes several 6-DOF constraints
- uses sequential convex programming

► Part of NASAs SPLICE project to modernize autonomous precision landing

- algorithm dev: UW, Draper, JSC
- HIL/flight demo: Blue Origin, Draper, JSC

▶ Plenty of room for improvement, both research-based and on the DQG implementation





Thank you!

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UW

M. Szmuk D. Malyuta M. Mesbahi B. Açıkmeşe

JSC

- J. M. Carson III
 - R. Sostaric
 - D. Matz E. Braden
- z T. Barrows n R. Loffi

Draper

J. Doll

M. Fritz

U. Lee



A Few Relevant Publications

Algorithm Development

- TPR, M. Szmuk, D. Malyuta, M. Mesbahi, B. Açıkmeşe and J. M. Carson III, "Dual Quaternion Based Powered Descent Guidance with State-Triggered Constraints," J. of Guidance, Control and Dynamics, vol. 43, no. 9, pp. 1584-1599, 2020
- M. Szmuk, TPR, and B. Açıkmeşe, "Successive Convexification for Real-Time 6-DoF Powered Descent Guidance with State-Triggered Constraints," *J. of Guidance, Control, and Dynamics*, vol. 43, no. 8, pp. 1399-1413, 2020
- TPR, D. Malyuta, M. Mesbahi, B. Açıkmeşe and J. M. Carson III, "A Real-Time Algorithm for Non-Convex Powered Descent Guidance," *AIAA SciTech Forum*, Orlando, FL. 2020
- D. Malyuta, TPR, M. Szmuk, M. Mesbahi, B. Açıkmeşe, and J. M. Carson III, "Discretization Performance and Accuracy Analysis for the Powered Descent Guidance Problem," *AIAA SciTech Forum*, Orlando, FL. 2019
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- R. Sostaric, S. Pedrotty, J. M. Carson III, et al., "The SPLICE Project: Safe and Precise Landing Technology Development and Testing," *AIAA SciTech Forum*, Virtual, 2021
- J. M. Carson III, M. M. Munk, R. Sostaric, et al., "The SPLICE Project: Continuing NASA Development of GN&C Technologies for Safe and Precise Landing," *AIAA SciTech Forum*, Orlando, FL. 2020