TQS 124 Autumn 2007 Quinn

Calculus & Analytic Geometry I

Functions Form the Foundation

What is a function?

A **function** is a rule that assigns to each element x (*called the input or independent variable*) in a set D exactly one element $f(x)$ (*called the ouput or dependent variable*) in a set E.

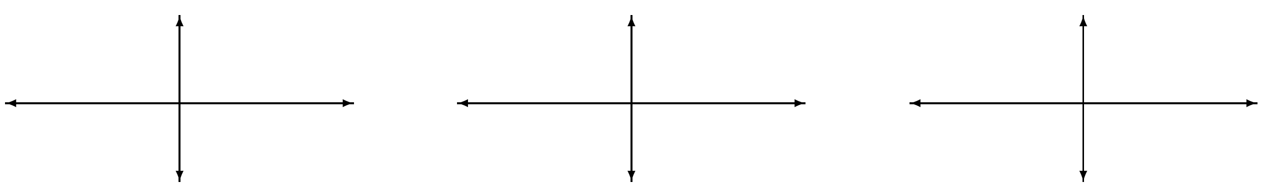
D: The set of possible values for the input variable is called the **domain.**

E: The set of all possible output values for $f(x)$ is called the **range.**

Note: The definition says *rule* and not *formula.* Why make this distinction?

Examples of functions:

Which of the following graphs represent functions and why (or why not)?



Linear Functions.

A function is **linear** if any change in the input causes a proportional change in the output.

So y is a linear function of x means that

any change in y	is proportional to	the corresponding change in x
Δy	$\propto$	Δx
Δy	$=$	$m\Delta x$

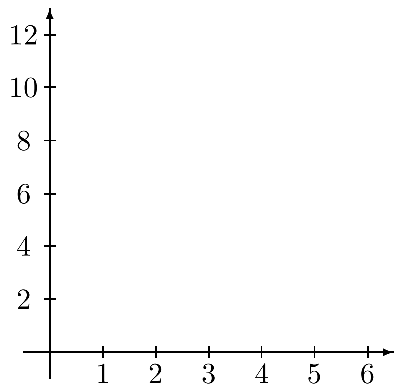
where m is the constant of proportionality.

Problem.

1. Suppose that y is a linear function of x. If $y = 2$ when $x = 0$ and $y = 5$ when $x = 2$, what is the constant of proportionality?

2. What is y when $x = 3$? when $x = 7$?

3. Plot the known values of y versus x on the graph below.



Formulas of a line.

The common formulas for a line can be derived from the equation of proportionality $\Delta y = m\Delta x$. Assume that (x_0, y_0) is a fixed point on the line, (x, y) is an arbitrary point, and m is the constant of proportionality. Then

$$\Delta y = m\Delta x$$

point slope form

initial value form

slope–intercept form

The constant m has many different interpretations.

$m \leftrightarrow$ slope
constant of proportionality
multiplier
rate of change of y with respect to x

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Essential Functions—Ghosts of Mathematics Past

- linear functions

- polynomial functions

- power functions

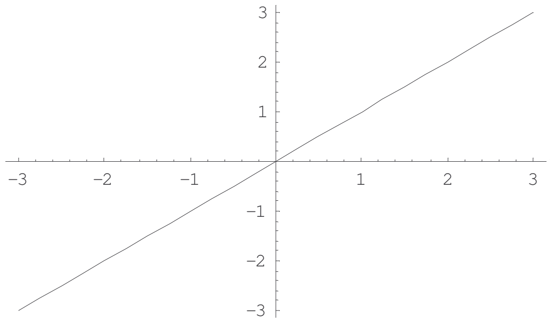
- rational functions

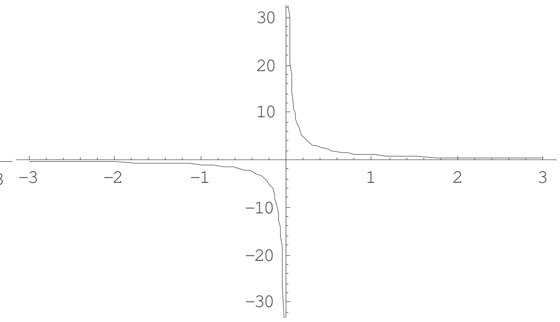
- trigonometric functions

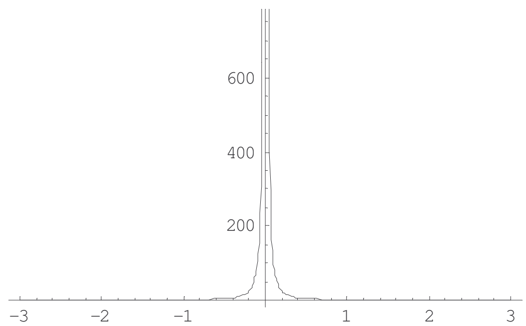
- exponential functions

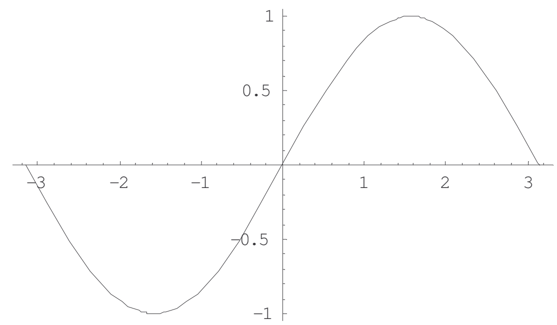
- logarithmic functions

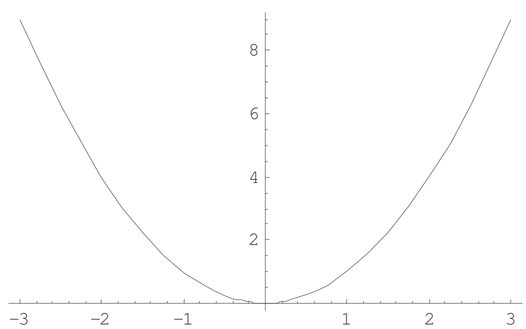
Make your best guess as to the function represented here. (e.g. Linear? Polynomial of degree n? Rational? Trigonometric? Exponential? Logarithmic?)

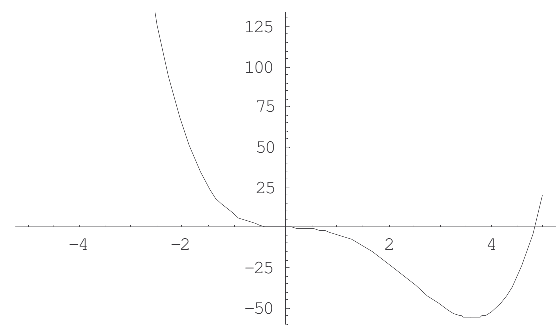


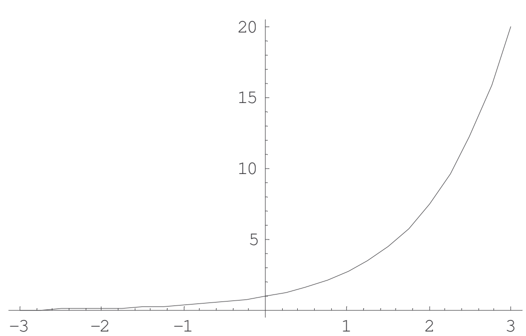


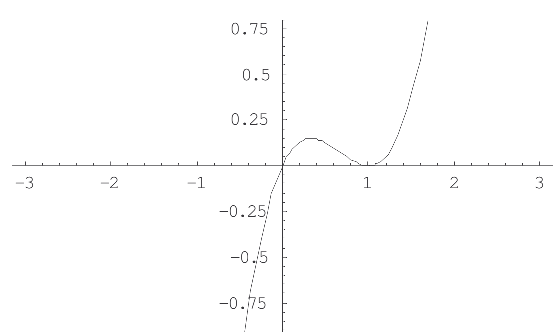












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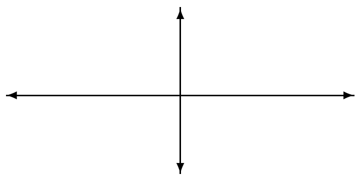
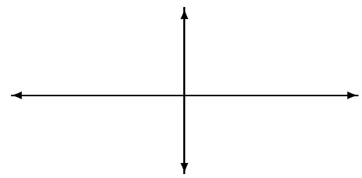
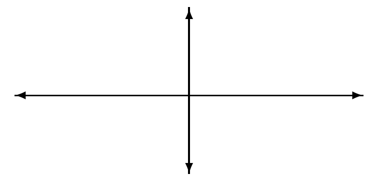
Making New Functions from Old Friends

—Shifting—

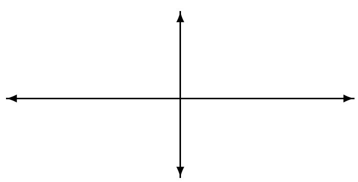
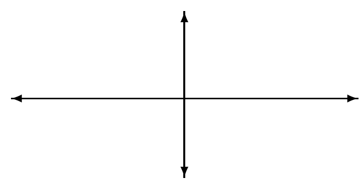
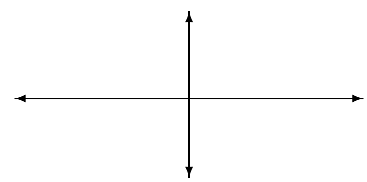
$f(x) = \ln(x)$ | $g(x) = \ln(x) + 2$ | $h(x) = \ln(x - 1)$

How would you change $f(x)$ to shift the graph down 3 units vertically?
How would you change $f(x)$ to shift its graph 2 units to the left?

—Stretching—

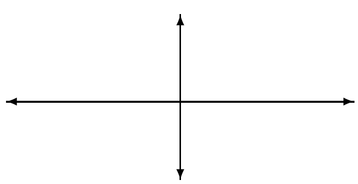
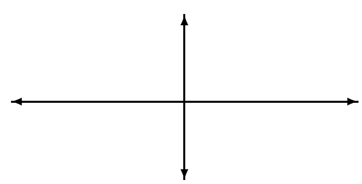
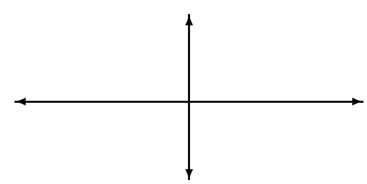
$f(x) = \cos(x)$ | $g(x) = 3\cos(x)$ | $h(x) = -2\cos(x)$

—Composition: Functions of functions—

The output of the function $f \circ g(x) = f(g(x))$ can be determined as by using the output of the function g as the input for the function f.

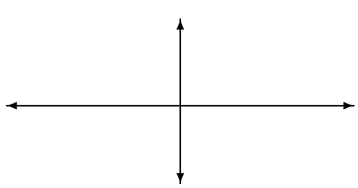
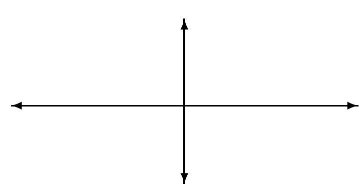
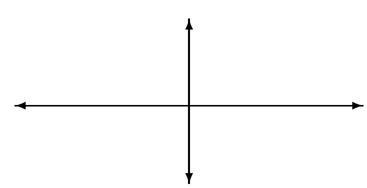
$f(x) = 2^x$ | $g(x) = x - 1$ | $h(x) = f \circ g(x)$

—Odd/ Even/ Neither—

A function $f(x)$ is called *odd* provided $f(-x) = -f(x)$ for all values of x. A function $f(x)$ is called *even* provided $f(-x) = f(x)$ for all values of x.

$f(x) = |x - 1|$ | $g(x) = \tan(x)$ | $h(x) = 2\cos(x)$

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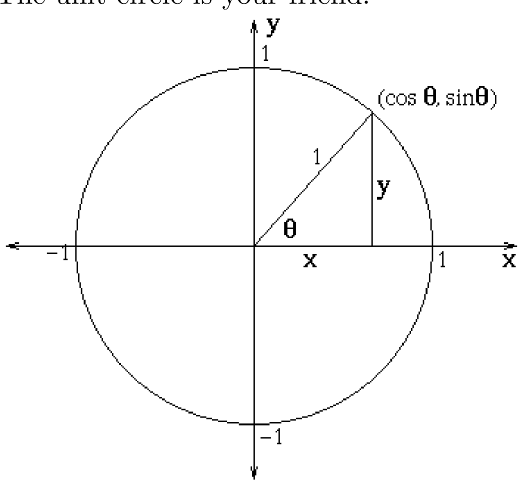
Brief Trig Encounter

In all likelyhood, we will not address this page in class. You are expected to be familiar with all trigonometric functions—especially the interpretation of the sine and cosine as the coordinates on a unit circle. I have provided this sheet for your benefit. You might want to work through it with friends (or come to office hours and discuss). Yes, you are expected to know this material.

—Shift,stretching review—
Sketch $f(x) = -2\sin(2x - \frac{\pi}{2})$ by successively graphing

$\sin(x)$
$\sin(2x)$
$\sin(2x - \frac{\pi}{2})$
$-2\sin(2x - \frac{\pi}{2})$.

Always work in radians! (radius units)
The unit circle is your friend.



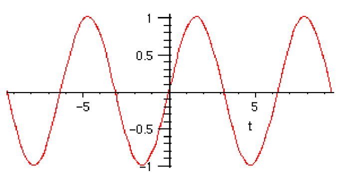
$\cos^2 x + \sin^2 x = 1$

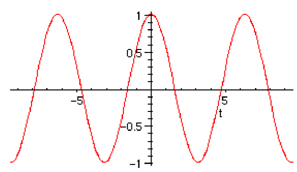
	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
0	$\sqrt{0}/2$	1	0
$\pi/6$	$\sqrt{1}/2$		
$\pi/4$	$\sqrt{2}/2$		
$\pi/3$	$\sqrt{3}/2$		
$\pi/2$	$\sqrt{4}/2$		
π	0	-1	0

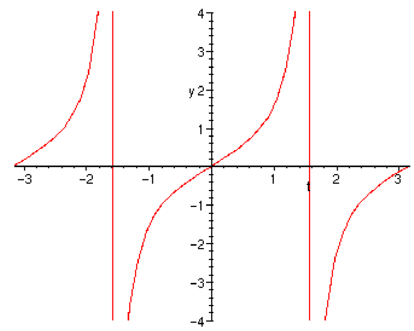
Quick Check

$\sin(5\pi/6)$ $\tan(4\pi/3)$ $\cos(13\pi/4)$

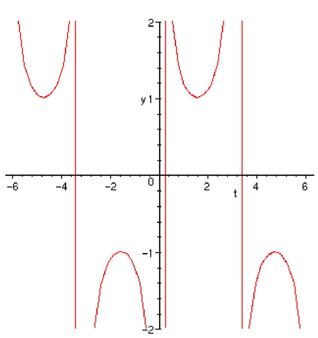
	$\sin(x)$	$\cos(x)$	$\tan(x)$
Domain :	x real $-\infty < x < \infty$	x real $-\infty < x < \infty$	x real $x \neq \pm\pi/2, \pm 3\pi/2, \ldots$
Range :	$-1 \leq x \leq 1$		
Period :			
Odd/Even?			
Sketch :			

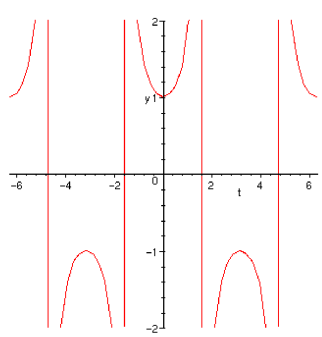


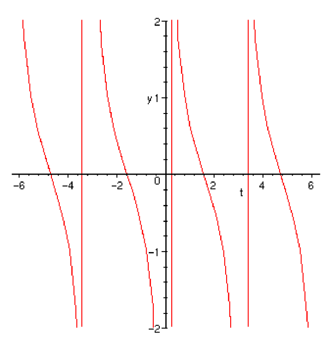




	$\csc(x)$	$\sec(x)$	$\cot(x)$
Domain :	x real	x real	x real
Range :			
Period :			
Odd/Even?			
Sketch :			







$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$