Calculus & Analytic Geometry I

Exponential Functions

Trouble with Tribbles. From Star Trek lore, we know that tribbles are fuzzy hermaphrodites about the size of an English muffin. Every month, a single tribble produces a litter of five offspring.

If we begin with one tribble at the beginning of the first month, how many tribbles are there at the end of the first month?

At the end of the second month?

At the end of the third month?

At the end of one year?

Estimate the number of years until the size of the tribble population is larger than the current human population of the earth. (According to the International Programs Center, U.S. Bureau of the Census, the total population of the World, projected to 9/28/07 at 23:32 GMT (EST+5) is 6,621,268,485.)

As you can see, we need to be good at manipulation exponential functions. Do you remember the following rules of exponents?

Review of Algebra		
$a^0 = 1$	$a^{-1} = 1/a$	$a^{-x} = \dfrac{1}{a^x}$
$a^x \cdot a^t = a^{x+t}$	$\dfrac{a^x}{a^t} = a^{x-t}$	$(a^x)^t = a^{xt}$

Question. Compare and contrast exponential functions to power functions?

If a function is said to have a doubling time/tripling time/half life/etc. that function must be exponential!

Question. Why do we make such a big deal about $e \approx 2.718281828...$? Can't we use any base (like 5 on the previous example)?

Sketch the shifted curves.

$y = e^x$ $\quad$ $y = e^{-x}$ $\quad$ $y = e^{x+1}$ $\quad$ $y = e^{-x-1}$

Simplify.

$9^{1/3} \cdot 9^{1/6}$ $\quad$ $2^{\sqrt{3}} \cdot 7^{\sqrt{3}}$ $\quad$ $\left(\dfrac{2}{\sqrt{2}}\right)^4$

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Inverse Functions and Logarithms

Unifying Idea An inverse of a function *undoes the action* of the function on it's input. So

$$\text{if } f(a) = b \text{ then } f^{-1}(b) = a.$$

Said another way, $f^{-1} \circ f(x) = x$ and $f \circ f^{-1}(x) = x$.

Examples. $f(x) = x + 2$ $\qquad g(x) = 5x$ $\qquad h(x) = \sqrt{x}$

Not every function has a well-defined inverse.

$f(x) = \sin(x)$ $\qquad g(x) = x^2$ $\qquad h(x) = 2$

Important Property. A function is *one-to-one* if $f(x_1) = f(x_2)$ implies that $x_1 = x_2$. In other words....

Are the following functions 1-1 on the given domains?

$\cos(x)$ on $[0, \pi]$ $\qquad \sin(x)$ on $[0, \pi]$ $\qquad \frac{1}{x}$ on all reals save $x = 0$.

If f is a 1-1 function on a domain B with range R, then f^{-1} exists and is a function from domain R to range B.

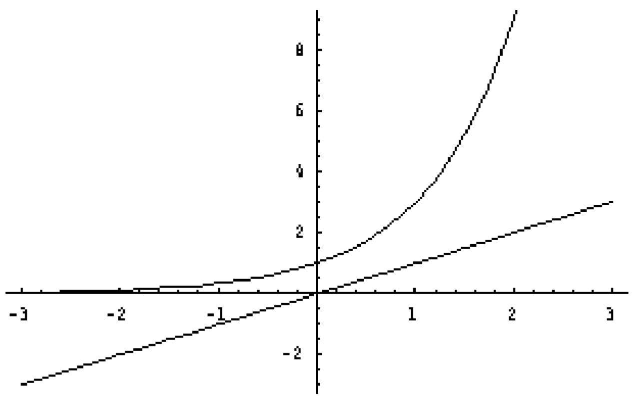
$$f : B \to R \qquad\qquad f^{-1} : R \to B$$

1. What is the domain of $f(x) = a^x$ (a a positive real number)?

2. Is $f(x) = a^x$ 1-1 on its domain? Why?

3. The inverse of f is defined to be $f^{-1} = \log_a(x)$.

$$y = a^x \qquad \Leftrightarrow \qquad \log_a(y) = x.$$



4. Simplify $5^{\log_5 7}$ $\qquad$ $\log_e(e^x)$ $\qquad$ $\log_{121}(11)$ $\qquad$ $2^{\log_4(x)}$.

Review of Algebra		
Using Exponents	Using Logarithms	General Information
$a^0 = 1$	$\log_c 1 = 0$	$\log_c x = y \leftrightarrow c^y = x$
$a^{-1} = 1/a$	$\log_c(AB) = \log_c(A) + \log_c(B)$	e is just a constant ≈ 2.71828
$a^{-x} = \dfrac{1}{a^x}$	$\log_c(A/B) = \log_c(A) - \log_c(B)$	$\log_e x = \ln x$
$a^x \cdot a^t = a^{x+t}$	$\log_c(A^p) = p \log_c(A)$	$c^t = (e^{\ln a})^t$
$\dfrac{a^x}{a^t} = a^{x-t}$	$\log_c(c^x) = x$	$\ln(e^x) = x$
$(a^x)^t = a^{xt}$	$c^{\log_c x} = x$	$e^{\ln x} = x$

Problem. The half life of a certain radioactive substance is 12 hours. There are 8 grams present initially.

- Express the amount of substance remaining as a funtion of time.
- When will there be 1 gram left?