Calculus & Analytic Geometry I

Speed, Tangent Lines, and Successive Approximation

Average vs. Instantaneous Speed. According to MapQuest, it should take 3 hours and 42 minutes to travel the 226 mi between the University of Washington, Tacoma and Oregon State University. What would be your average speed on this trip? How does that differ from the information on your speedometer?

speed vs. velocity?

Estimating the rate of change.

The *average rate of change of a function* $y = f(x)$ *on an interval* $[a, b]$ is given by the change in the output divided by the change in the input:

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in output}}{\text{change in input}} = \frac{f(b) - f(a)}{b - a} = \frac{f(a + h) - f(a)}{h}$$

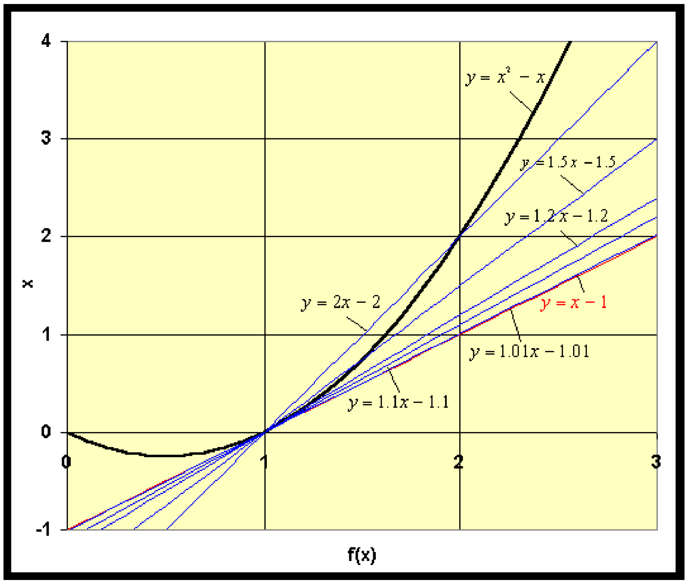
if $h = b - a$.

What is the average rate of change of the function $f(x) = 3x + 2$ on the interval $[4, 10]$.

For a linear function, the rate of change (slope of the line) equals the average rate of change. For other functions, an average rate of change on an interval is used to *estimate* the rate of change of the function at a point inside that interval. The instantaneous rate of change gives the slope of the *tangent line* to the curve at a point.

Problem. Approximate the average rate of change for the function $f(x) = x^2 - x$ at the point $P(1, 0)$.

$[1, 2]$ $[1, 1.5]$ $[1, 1.1]$



As the interval of consideration shrinks, the distance between the points of intersection of the curve and the secant line ...

Definition. The *slope of a graph at a point* is the limit of the slopes seen in a microscope at that point, as the field of view shrinks to zero. (Also called the *rate of change of a function at a point.*)

Problem. Given the function

$$f(x) = \frac{2 + x^3 \cos(x) + 1.5^x}{x + x^2}$$

estimate the *rate of change* of the function at $x = 2$.

Given h, calculate $(f(2+h) - f(2))/h$.

Example. $h = 0.1$.

$$\frac{f(2.1) - f(2))}{0.1} = \frac{-0.5104076048... - .153470884604...}{.1} \approx -6.639$$

So what is a LIMIT?

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Limits—Intuitively Thinking

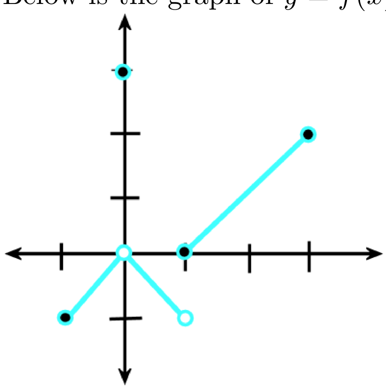
The *limit of a function $f(x)$ at a point c* is M, if the value of the function gets arbitrarily close to M for inputs close to c.

$$\lim_{x\to c} f(x) = M$$

Examples.

$$\lim_{x\to 3} x^2 - 3 \qquad \lim_{x\to 1} \frac{x^3-1}{x-1} \qquad \lim_{x\to 0} \frac{\sin x}{x}$$

Below is the graph of $y = f(x)$.



Find the following limits if they exist:

$$\lim_{x\to 0} f(x) \qquad \lim_{x\to 1} f(x)$$

To have a limit at a point c, a function f must be defined on <u>both</u> sides of c. However we can consider the functions behavior from either side individually

$$\lim_{x\to c^+} f(x) = L \qquad \lim_{x\to c^-} f(x) = M$$

$$\lim_{x\to -1^-} f(x) \qquad \lim_{x\to 0^-} f(x) \qquad \lim_{x\to 1^-} f(x) \qquad \lim_{x\to 3^-} f(x)$$

$$\lim_{x\to -1^+} f(x) \qquad \lim_{x\to 0^+} f(x) \qquad \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 3^+} f(x)$$

Theorem. A function $f(x)$ has a limit as x approaches c if and only if it has a left-handed and right-handed limit at c and these one-sided limits are equal.

Problem. Find $\lim_{z\to 3^+} \frac{\lfloor z \rfloor}{z}$ and $\lim_{z\to 3^-} \frac{\lfloor z \rfloor}{z}$

Vertical Asymptotes. Can a limit ever be infinite?

Let's see what happens as we approach the *holes* in the domain from the left and the right for

$$g(x) = \frac{e^x}{x^2} \qquad\qquad h(x) = \frac{2x^2 - x - 1}{3x^3 + 2x^2 - 5x}.$$

Writing $\lim_{x\to c^+} f(x) = \infty$ **does not mean** that ∞ is a number. It **does not mean** that the limit exists. It **means** that the *value of $f(x)$ becomes arbitrarily large as x approaches c from the right.*

If $\lim_{x\to c^+} f(x) = \infty$ and $\lim_{x\to c^-} f(x) = \infty$, we will go so far as to say

$$\lim_{x\to c} f(x) = \infty.$$

Similar meaning for $\lim_{x\to c} f(x) = -\infty$