Calculus & Analytic Geometry I

Horizontal Asymptotes

Limits at positive or negative infinity tell us how the function "eventually" behaves (if ever)...

$f(x) = e^{-x}$ $\qquad$ $g(t) = \frac{7t^3}{t^3-3t^2+6t}$ $\qquad$ $h(z) = \frac{2+\sqrt{z}}{2-\sqrt{z}}$

$f(x) = \frac{2x^3-2x+3}{3x^3+2x^2-5x}$ $\qquad$ $h(x) = \frac{2x^2-2x+3}{3x^3+2x^2-5x}$ $\qquad$ $g(x) = \frac{2x^4-2x+3}{3x^3+2x^2-5x}$

A line $y = b$ is a *horizontal asymptote* of the graph of a function $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = b \qquad \text{or} \qquad \lim_{x\to-\infty} f(x) = b.$$

Problem. Find the asymptotes (horizontal and vertical) for $f(x) = \frac{x^3+7x^2-x-7}{x^2+x-2}$.

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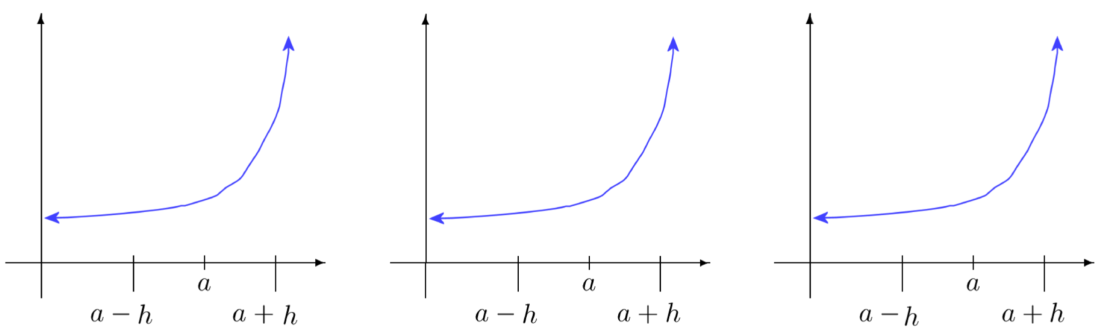
Tangents and Derivatives at a Point

Informally, the *derivative* of a function $y = f(x)$ at a point $x = a$ is defined to be the slope (or the rate of change) of the function at the point $(a, f(a))$. In §2.1, we estimated the rate of change (a.k.a. the derivative) by calculating the average rates of change of the function on smaller and smaller intervals containing a and watched the rates stabilize.

For each of the intervals given below, write an equation representing the average rate of change for the general function $y = f(x)$ on the specified interval. Then, on the graph provided sketch the line used to calculate the average rate of change on that interval.

Interval:	$(a-h, a)$	$(a, a+h)$	$(a-h, a+h)$
Avg. rate of change:			



The expressions you have just discovered are referred to as <u>difference quotients</u>. *Each expression* acts as an estimate for the derivative $f'(a)$. To improve these estimates, let $h \to 0$. This leads us to the formal definition of the derivative.

Definition. The *derivative (slope, rate of change)* of a function f at $x = a$ is defined as:

$$\begin{aligned} f'(a) &= \lim_{h\to 0} \frac{f(a) - f(a-h)}{h} \\ &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h\to 0} \frac{f(a+h) - f(a-h)}{2h} \end{aligned}$$

provided these limits exist and are equal. In this case, f is said to be *differentiable* at $x = a$.

Are all three difference quotients really needed? Consider $f(x) = |x|$ at $x = 0$.

Problems. Find the slope of the function's graph at the given point. Then find the equation for the line tangent to the graph there.

1. $y = 2\sqrt{x}, \qquad (1, 2)$

2. $y = \dfrac{x-1}{x+1}, \qquad x = 0$

Summary. The following ideas are equivalent:

- the slope of $y = f(x)$ at $x = x_0$
- the slope of the tangent to the curve $y = f(x)$ at $x = x_0$
- the rate of change of $f(x)$ with respect to x at $x = x_0$
- the derivative $f'(x_0)$
- the limit of (any) difference quotient, $\lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$