CALCULUS & ANALYTIC GEOMETRY I

The Extremes

We are beginning Chapter 4: Applications of Differentiation. Today we examine the maximum and minimum values of functions (a.k.a. the extremes).

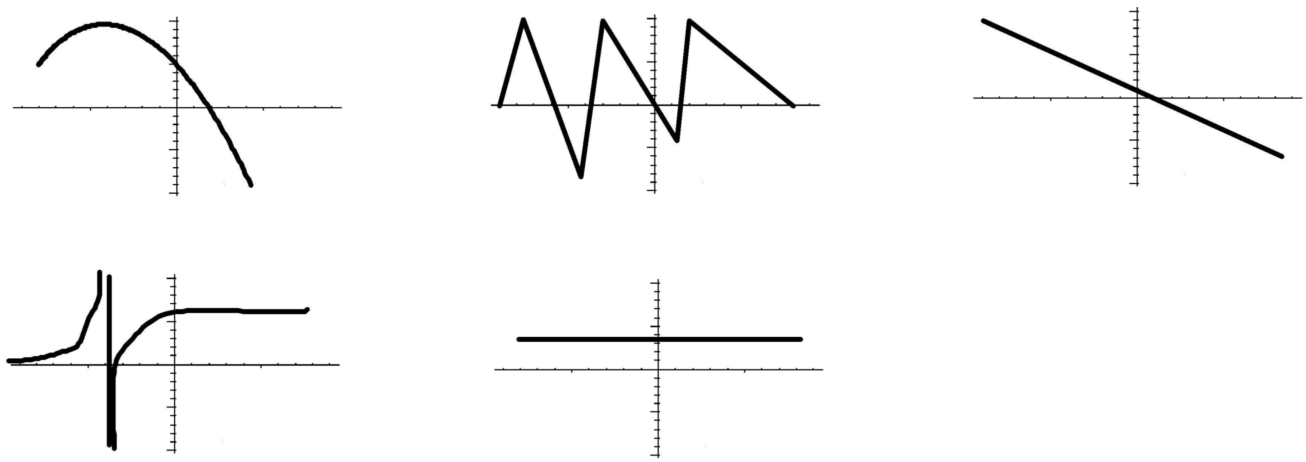
Definition. A function f with Domain D has an *absolute (global) maximum* at c if

$$f(x) \leq f(c) \text{ for all } x \text{ in } D$$

and an *absolute (global) minimum* at c if

$$f(x) \geq f(c) \text{ for all } x \text{ in } D.$$

The extreme value theorem says that a continuous function on a closed interval always has both an absolute maximum M and an absolute minimum m.



Definition. A function f with Domain D has a *local maximum* at c if

$$f(x) \leq f(c) \text{ for all } x \text{ in some open interval containing } c$$

and an *local minimum* at c if

$$f(x) \geq f(c) \text{ for all } x \text{ in an open interval containing } c.$$

What characteristics do extremes have?

Definition. A *critical point* is a point in the of the domain of a function f where either

- f' is zero, or
- f' is undefined.

Critical points are candidates for local extremes. Critical points and endpoints are candidates for global extremes.

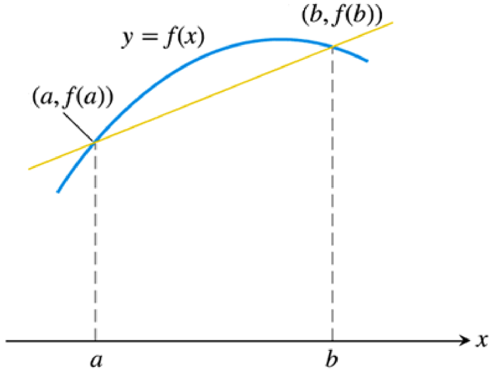
Problems.

1. Find the absolute maximum and minimum value for $g(x) = xe^{-x}$ on the interval $-1 \leq x \leq 1$.

2. Find the absolute maximum and minimum value for $h(t) = 2 - |t|$ on the interval $-1 \leq t \leq 3$.

The Theorems—Most Especially the MVT

The Intermediate Value Theorem. A function $y = f(x)$ that is continuous on $[a, b]$ takes on every value between $f(a)$ and $f(b)$.



Rolle's Theorem. If a function $y = f(x)$ is

- continuous on $[a, b]$
- differentiable on (a, b)
- $f(a) = f(b)$

then there exists a c in (a, b) such that $f'(c) = 0$.

The Mean Value Theorem. If a function $y = f(x)$ is

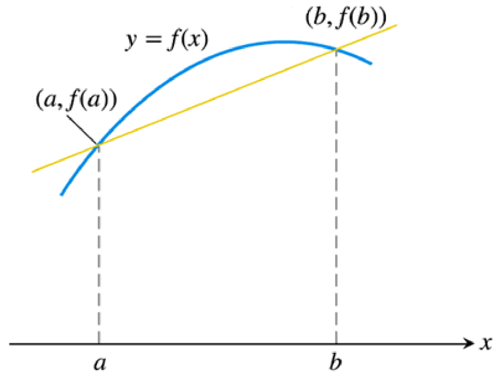
- continuous on $[a, b]$
- differentiable on (a, b)

then there exists a c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Proof of Rolle's Theorem.

Application of Rolle's Theorem. Show that the function $f(x) = \frac{1}{1-t} + \sqrt{1+t} - 3.1$ has exactly one real zero in the interval $(-1, 1)$.

Proof of the MVT.



Let $h(x) = f(x) - ($equation of secant line $)$.

Corollary 1. If $f'(x) = 0$ at each point of (a, b), then $f(x)$ is a constant function on the interval (a, b). (i.e. $f(x) = C$ for some real number C.)

Corollary 2. If $f'(x) = g'(x)$ at each point of (a, b), then $f(x)$ and $g(x)$ differ by a constant function on the interval (a, b). (i.e. $f(x) = g(x) + C$ for some real number C.)

Problems.

1. Find all possible function $f(x)$ such that $f'(x) = x^3$.
2. Find the function $g(t)$ such that $g'(t) = e^{2t}$ and $g(0) = \frac{3}{2}$.
3. Suppose the acceleration of a body is measure as 9.8 m/s^2 and we know that $v(0) = -3$ m/s, $s(0) = 5$ m. Find an equation to describe the body's position at time t.