Calculus & Analytic Geometry I

Indeterminant Forms and l'Hôpital's Rule

Application of derivatives to assess pesky limits...

Indeterminant Forms. Sometimes we need to evaluate $\lim_{x\to c}\dfrac{f(x)}{g(x)}$ where $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ are either both 0 or both ∞ ($\frac{0}{0}$ or $\frac{\infty}{\infty}$).

Example. $\lim_{x\to 2}\dfrac{x^7-128}{x^3-8}$

l'Hôpital's Rule. Suppose that $f(c)=g(c)=0$ and that f and g are differentiable on an open interval I containing c, and that $g'(x)\neq 0$ on I if $x\neq c$. Then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)},$$

assuming that the limit on the right side of this equation exists.

Problems. Verify the following limits:

1. $\lim_{x\to 0}\dfrac{\sin x}{x}=1$
2. $\lim_{x\to 0}\dfrac{x-\sin x}{x^3}=\dfrac{1}{6}$
3. $\lim_{x\to 0}\dfrac{1-\cos x}{\sec x}=0$
4. $\lim_{x\to\infty}\dfrac{2x^2-3x+1}{3x^2+5x-2}=\dfrac{2}{3}$
5. $\lim_{x\to\infty}\dfrac{x+\sin x}{x-\cos x}=1$
6. $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$
7. $\lim_{x\to 0+} x^{\sin x}=1$

The Indeterminant Forms: $\dfrac{0}{0}$ $\quad\dfrac{\infty}{\infty}$ $\quad 0\cdot\infty$ $\quad\infty-\infty$ $\quad 0^0$ $\quad\infty^0$ $\quad 1^\infty$

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Curve Sketching

Problem. Sketch the general shape of a curve satisfying the given information

interval:	$x < 0$	$0 < x < 2$	$2 < x < 3$	$3 < x$
sign of f' :	$-$	$-$	$-$	$+$
sign of f'' :	$+$	$-$	$+$	$+$

Strategies for Graphing Functions

- Identify domain and any symmetries the curve may have.
- Find first and second derivatives.
- Find critical points and identify behavior at each.
- Determine where function is increasing or decreasing.
- Find points of inflection and concavity.
- Identify asymptotes (l'Hôpital may come in handy).
- Plot key points (intercepts and anything found above).

More Problems. Graph as many of the following functions as time will permit.

1. $y = 4x^3 - x^4$
2. $y = 2x - 3x^{2/3}$
3. $y = e^{2/x}$
4. $y = \dfrac{(x+1)^2}{1+x^2}$