Calculus & Analytic Geometry I

Antiderivatives and Initial Value Problems

Definition. A function F is an *antiderivative* of f on an interval I if $F'(x) = f(x)$.

Question. Given a nice function f, how many antiderivatives can it have?

Definition The set of all antiderivatives of f is the *indefinite integral* of f with respect to x, denoted

$$\int f(x)dx.$$

The symbol $\int$ is an *integral sign.* The function f is the *integrand* and x is the *variable of integration.*

Theme. Every differentiation problem corresponds to an antidifferentiation problem.

Differentiation Problems	Antidifferentiation Problems
$(x^2)' \; = \; 2x$	$\int 2x dx \; = \; x^2 + C$
$=$	$\int \cos(x) dx \; =$
$(e^x + \ln(x))' \; =$	$=$
$(\cos(x^2))' \; =$	$=$
$(e^{\tan(x)})' \; =$	$=$
$=$	$\int 4x \sin(x^2) dx \; =$
$=$	$\int \cos(x) e^{\sin(x)} \; =$

A *differential equation* is an equation that involves a function and its derivatives. An **initial value problem (IVP)** asks you to solve for a *particular* antiderivative based on a differential equation and an initial condition.

1. Find $s(t)$ if $\frac{ds}{dt} = \cos t + \sin t,\ s(\pi) = 1.$

2. Find $v(x)$ if $\frac{dv}{dx} = \frac{1}{2} \sec x \tan x,\ v(0) = 1.$

3. Find $y(t)$ if $\frac{d^2y}{dt^2} = \frac{3t}{8},\ \left.\frac{dy}{dt}\right|_{t=1} = 3$, and $y(4) = 4.$

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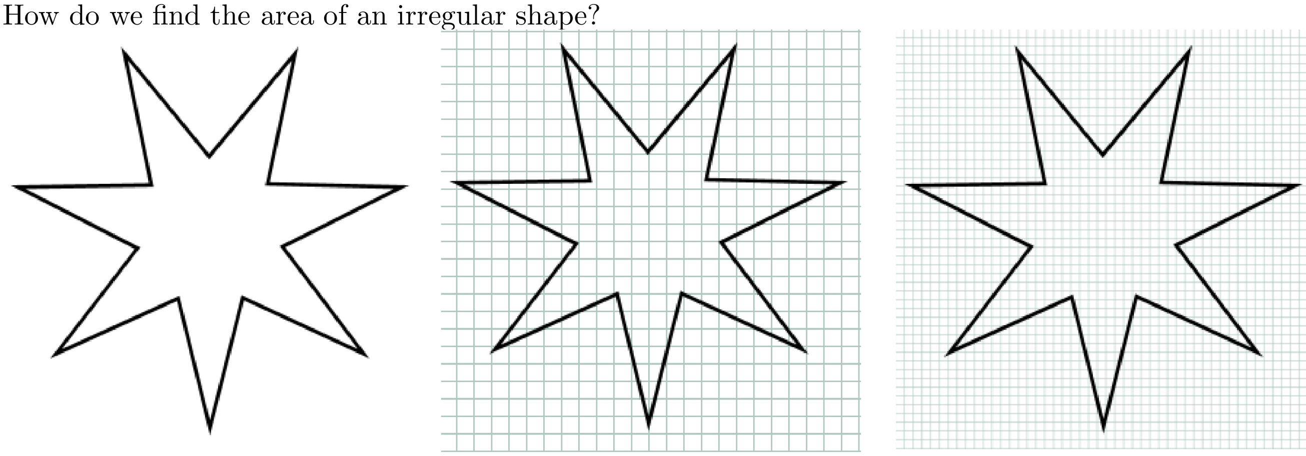
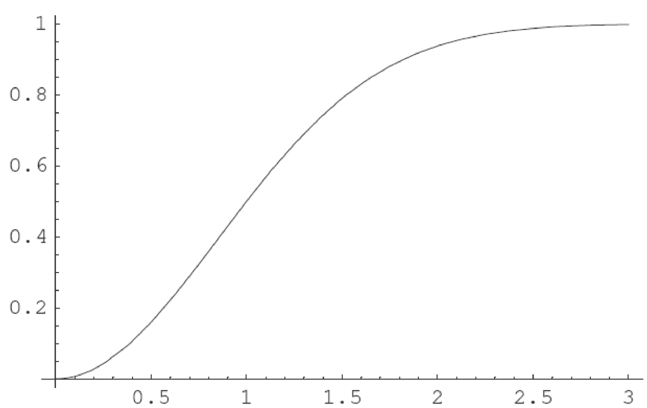
Subdivide–Approximate–Accumulate–Refine

How do we find the area of an irregular shape?



The same will be true for finding areas under curves, distance traveled, and average values of functions.

Area under a curve. Let's approximate the area under the curve $f(x) = 1 - 2^{-x^2}$ between $0 \leq x \leq 3$.



Number of subdivisions: $n =$

$\Delta x =$

lower sum:

upper sum:

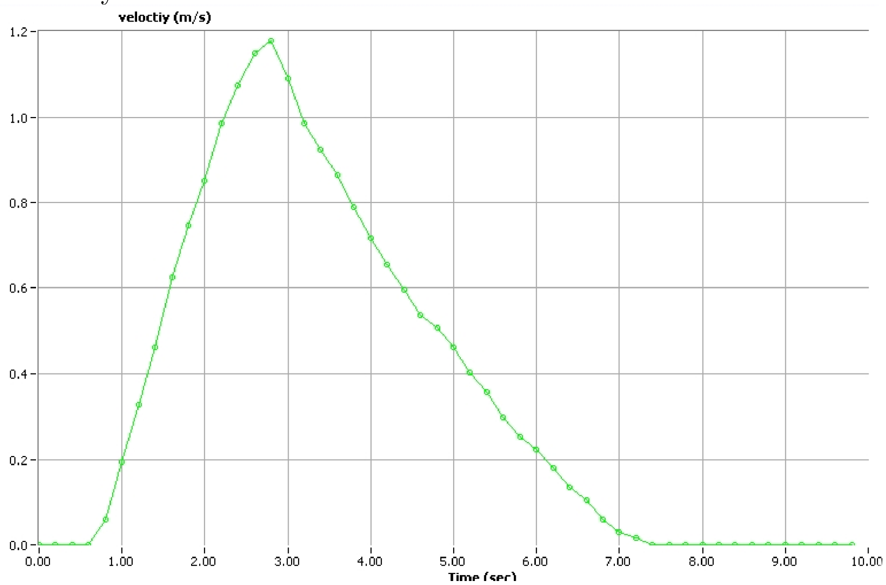
midpoint rule:

To improve our approximation, increase the number of subintervals. (A programmable calculator comes in handy here. If you have a TI-83 or 84, I recommend http://math.ucsd.edu/~ashenk/Calculators/Riemann_TI-83.pdf.)

n	lower sum	upper sum	midpoint rule
10	1.78632...	2.08573...	1.93594...
50	1.90603...	1.96591...	1.93597...
100	1.92100...	1.95094...	1.93597...
250	1.93000...	1.94196...	1.93597...

Distance Traveled. Exact same process—previously we accumulated approximations for area, now we accumulate approximations for distance traveled (based on velocity.)

The following data was collected from a matchbox car traveling down a ramp. Estimate how far the car toy traveled.



What was the average velocity of the toy during the period that is was moving?

In general, the *average value* of a continuous function $f(x)$ on an interval $[a, b]$ is the area under the curve divided by the length of the interval.