Sigma notation and Definite Integrals

$$\sum_{k=1}^{n} a_k$$

Illustrations.

- $\displaystyle\sum_{k=1}^{3} \frac{k-1}{k}$
- $\displaystyle\sum_{i=-1}^{4} 3 \cdot 2^{k+1}$
- $\displaystyle\sum_{j=0}^{2} \frac{(-1)^j}{j+1}$
- $\displaystyle\sum_{j=-1}^{1} \frac{(-1)^j}{j+2}$
- $\displaystyle\sum_{k=3}^{10} 1$
- $\displaystyle\sum_{i=1}^{10} n+i$

Rules for Finite Sums

1. Sum Rule: $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$.
2. Difference Rule: $\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$.
3. Constant Multiple Rule: $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$.
4. Constant Value Rule: $\sum_{k=1}^{n} c = n \cdot c$.

Some Important Sums

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Let $f(x) = 1 - x + 2x^2$. Find a formula for the upper sum obtained by dividing the interval $[1, 3]$ into n equal subintervals. Then take the limit of these sums as $n \to \infty$ to calculuate the area under curve on $[0, 3]$.

Riemann Sum. For a function $f(x)$ on an interval $[a, b]$ the Riemann Sum

$$\sum_{k=0}^{n} f(x_i^*)\Delta x$$

approximates the (signed) area under the curve from $[a, b]$ using n intervals.

$\Delta x=$
Endpoints $a = x_0, x_1, x_2, \ldots, x_{n-1}, x_n = b$:
Sample points x_i^* lies in ith subinterval $[x_{i-1}, x_i]$

The definite integral of f over $[a, b]$ is the limit of the Riemann sum $\sum_{k=0}^{n} f(x_i^*)\Delta x$ as $n \to \infty$ using *any* choice of x_i^* in $[x_{i-1}, x_i]$, provided the limit exists. If the limit exists, we say the function is *integrable on* $[a, b]$.

$$\int_a^b f(x)dx$$

A continuous function is always integrable, that is to say

Compute $\int_0^b cdx$ where c is a fixed real number.

Compute $\int_0^b x^2dx$ where c is a fixed real number.

Properties of Definite Integrals

1. Order of Integration: $\int_a^b f(x)dx = -\int_b^a f(x)dx$

2. Zero Width Interal: $\int_a^a f(x)dx =$

3. Constant Multiple: $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for any number k

   $\int_a^b -f(x)dx = -\int_a^b f(x)dx$

4. Sum and Difference: $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

5. Additivity: $\int_a^b f(x)dx + \int_b^c f(x)dx = \int$

6. Max-Min Inequality: If f attains a maximum and minimum value on the interval $[a, b]$ then

$$\min f \cdot (b-a) \leq \int_a^b f(x)dx \leq \max f \cdot (b-a)$$

7. Domination: If $f(x) \geq g(x)$ on $[a, b]$ then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

Suppose that $\int_1^2 f(x)dx = -4,$ $\int_1^5 f(x)dx = 6,$ and $\int_1^5 g(x)dx = 8.$ Find

$\int_2^5 f(x)dx$ $\int_1^5 [4f(x) - g(x)]dx$ $\int_2^2 f(x)dx$ $\int_5^1 [g(x) - f(x)]dx$