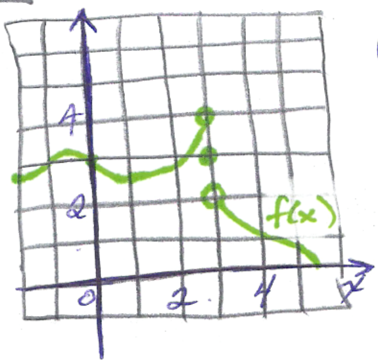
2.2.4 STATE VALUE OF EACH QUANTITY, IF IT EXISTS.



(a) $\lim_{x \to 0} f(x) = 3$

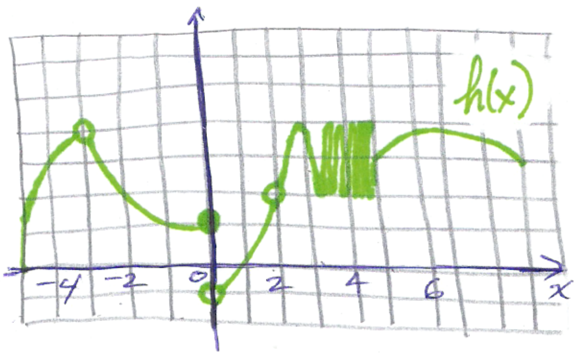
(b) $\lim_{x \to 3} f(x) \neq 4$

(c) $\lim_{x \to 3^+} f(x) = 2$

d) $\lim_{x \to 3} f(x)$ D.N.E
since limit from below and above differ

e) $f(3) = 3$
[Asking for value of function]

2.2.6



(b) $\lim_{x \to 3^+} h(x) \cong 4$ (THE VALUE OF THE FUNCTION)

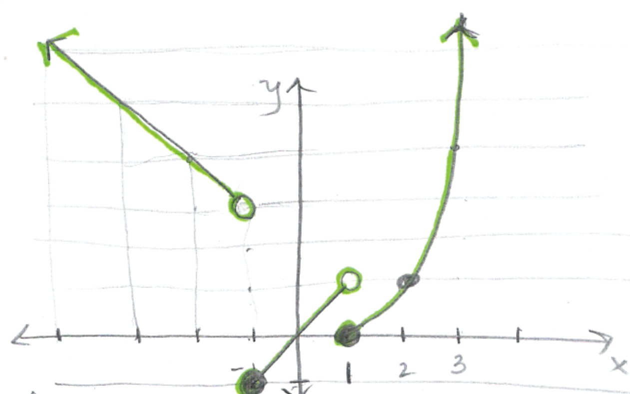
(e) $\lim_{x \to 0^-} h(x) = 1$

(h) $h(0) = 1$

(k) $\lim_{x \to 5^+} h(x) = 3$ (NOTE $\lim_{x \to 5^-} h(x)$ D.N.E)

2.2.12 Sketch graph of f(x) and determine value a for which $\lim_{x \to a} f(x)$ exists

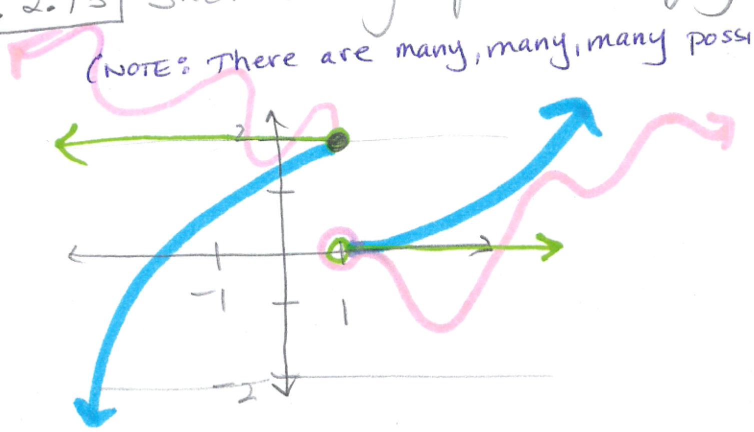
$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ (x-1)^2 & \text{if } x \ge 1 \end{cases}$$



f(x) is continuous everywhere except for x = −1 and x = 1.

2.2.13 Sketch a graph satisfying $\lim_{x \to 1^-} f(x) = 2$, $\lim_{x \to 1^+} f(x) = -2$, AND $f(1) = 2$

(NOTE: There are many, many, many possible correct answers!)



EACH COLOR REPRESENTS A CORRECT YET DIFFERENT SOLUTION.

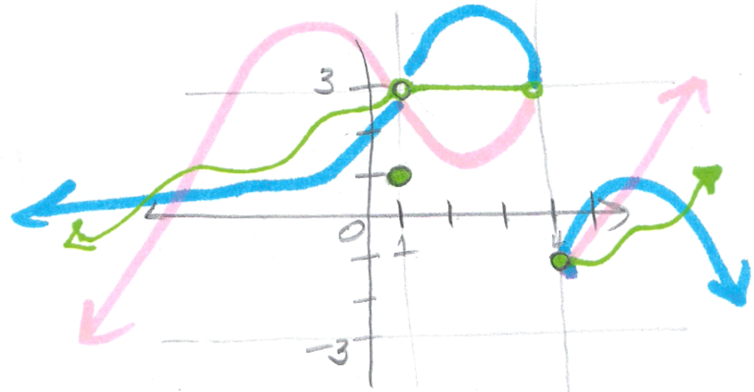
2.2.16 Sketch a graph satisfying

$\lim_{x\to 1} f(x) = 3$ $f(1) = 1$

$\lim_{x\to 4^-} f(x) = 3$ $f(4) = -1$

$\lim_{x\to 4^+} f(x) = -3$

(AGAIN LOTS OF POSSIBILITIES GIVEN IN DIFFERENT COLORS)



2.2.19 Guess limit based on requested data.

$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

x	$\frac{e^x-1-x}{x^2}$
1	.71828183...
.5	.59488508...
.1	.51709180...
.05	.50843855...
.01	.5016708....

x	$\frac{e^x-1-x}{x^2}$
-1	.3678794...
-.5	.42612263...
-.1	.48374180...
-.05	.49176980...
-.01	.49833749...

IT VERY MUCH LOOKS LIKE $\lim_{x\to 0} \frac{e^x-1-x}{x^2} = .5$

2.2.27 Determine infinite limit: $\lim_{x\to 1} \frac{2-x}{(x-1)^2}$

At $x=1$, $\frac{2-x}{(x-1)^2}$ has the form "$\frac{1}{0}$", so the limit will be infinite (as they told us in the statement of the problem. We need to determine whether it is going to positive or negative ∞.

Consider the one sided limits.

$\lim_{x\to 1^-} \frac{2-x}{(x-1)^2}$ and $\lim_{x\to 1^+} \frac{2-x}{(x-1)^2}$.

If $x \to 1^-$, the factor $(2-x)$ is positive while the denominator is always positive So $\lim_{x\to 1^-} \frac{2-x}{(x-1)^2} = +\infty$

If $x\to 1^+$, the factor $(2-x)$ is negative while the denominator remains positive So $\lim_{x\to 1^+} \frac{2-x}{(x-1)^2} = -\infty$.

Since the limit from the left and right do not agree, we conclude that $\lim_{x\to 1} \frac{2-x}{(x-1)^2}$ does not exist.

2.2.28 Determine the infinite limit $\lim\limits_{x \to 5^-} \frac{e^x}{(x-5)^3}$

As $x \to 5^-$, the function have to form "$\frac{e^5}{0}$" Again the answer will be either positive or negative infinity.

As $x \to 5^-$: e^x approaches e^5 and is always positive.
$(x-5)$ will be a small (in magnitude), negative number. So $(x-5)^3$ remains negative.

Since $\frac{+}{-} = -$ we conclude

$$\lim_{x \to 5^-} \frac{e^x}{(x-5)^3} = -\infty$$