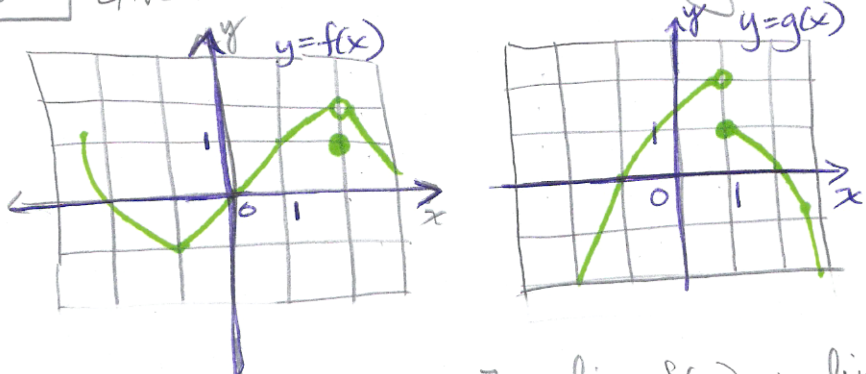
$\boxed{2.3.1}$ Given $\lim_{x\to 2} f(x) = 4$, $\lim_{x\to 2} g(x) = -2$, and $\lim_{x\to 2} h(x) = 0$. Find

(a) $\lim_{x\to 2}[f(x)+5g(x)] = \lim_{x\to 2} f(x) + 5\lim_{x\to 2} g(x) = 4+5\cdot(-2) = -6$

(c) $\lim_{x\to 2}\sqrt{f(x)} = \sqrt{\lim_{x\to 2} f(x)} = \sqrt{4} = 2$

(e) $\lim_{x\to 2}\frac{g(x)}{h(x)} \underset{\text{(if it exists)}}{=} \frac{\lim_{x\to 2} g(x)}{\lim_{x\to 2} h(x)} = \frac{-2}{0}$. Oops. This limit does not exist because we are dividing by zero.

$\boxed{2.3.2}$ Given the information in the graphs, Evaluate given limits.



(a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2 + 0 = 2$

(c) $\lim_{x\to 0}[f(x)g(x)] = \left[\lim_{x\to 0} f(x)\right]\cdot\left[\lim_{x\to 0} g(x)\right] = 0\cdot 1.4 = 0$

(e) $\lim_{x\to 2}[x^3 f(x)] = \left[\lim_{x\to 2} x^3\right]\cdot\left[\lim_{x\to 2} f(x)\right] = 2^3\cdot 2 = 2^4$.

$\boxed{2.3.4}$ Evaluate (and justify each step)

$$\lim_{x\to 2}\frac{2x^2+1}{x^2+6x-4} = \frac{\lim_{x\to 2}(2x^2+1)}{\lim_{x\to 2}(x^2+6x-4)} \quad \text{BY QUOTIENT RULE}$$

$$= \frac{\lim_{x\to 2}(2x^2) + \lim_{x\to 2}(1)}{\lim_{x\to 2}(x^2) + \lim_{x\to 2}(6x) + \lim_{x\to 2}(-4)} \quad \text{By SUM RULE}$$

$$= \frac{2\lim_{x\to 2}(x^2) + \lim_{x\to 2}(1)}{\lim_{x\to 2}(x^2) + 6\lim_{x\to 2} x + \lim_{x\to 2}(-4)} \quad \text{BY CONSTANT MULTIPLE RULE}$$

$$= \frac{2\left[\lim_{x\to 2}(x)\right]^2 + \lim_{x\to 2}(1)}{\left[\lim_{x\to 2}(x)\right]^2 + 6\lim_{x\to 2} x + \lim_{x\to 2}(-4)}$$ BY POWER LAW

$$= \frac{2[2]^2 + 1}{2^2 + 6\cdot 2 - 4}$$ BY CONSTANT FUNCTION & IDENTITY FUNCTION RULES (#7 AND #8).

$$= \frac{5}{12}.$$

$\boxed{2.3.5}$ Same directions as 4 above.

$$\lim_{x\to 8}(1+\sqrt[3]{x})(2-6x^2+x^3) = \lim_{x\to 8}(1+\sqrt[3]{x}) \cdot \lim_{x\to 8}(2-6x^2+x^3)$$ PRODUCT RULE

$$= \left(\lim_{x\to 8} 1 + \lim_{x\to 8}\sqrt[3]{x}\right)\cdot\left(\lim_{x\to 8} 2 - 6\lim_{x\to 8} x^2 + \lim_{x\to 8} x^3\right)$$ SUM RULE + CONST. MULT RU

$$= \left(\lim_{x\to 8} 1 + \sqrt[3]{\lim_{x\to 8} x}\right)\left(\lim_{x\to 8} 2 - 6\left(\lim_{x\to 8} x\right)^2 + \left(\lim_{x\to 8} x\right)^3\right)$$ POWER RULE.

$$= (1+\sqrt[3]{8})(2-6(8)^2+8^3)$$ BY CONST. IDENTITY + IDENTITY FUNCTION RULES

$$= 3(130) = 390$$

$\boxed{2.3.8}$ $$\lim_{u\to -2}\sqrt{u^4+3u+6} = \sqrt{\lim_{u\to -2}(u^4+3u+6)}$$ (FRACTIONAL) POWER LAW

$$= \sqrt{\lim_{u\to -2} u^4 + 3\lim_{u\to -2} u + \lim_{u\to -2} 6}$$ SUM LAW AND CONSTANT MULTIPLE LAW.

$$= \sqrt{\left(\lim_{u\to -2} u\right)^4 + 3\left(\lim_{u\to -2} u\right) + \lim_{u\to -2} 6}$$ POWER LAW

$$= \sqrt{(-2)^4 + 3(-2) + 6}$$ BY CONSTANT FUNCTION AND IDENTITY FUNCTION LAWS

$$= \sqrt{16} = 4.$$ PHEW!

$\boxed{2.3.19}$ $\lim_{x\to -2} \frac{x+2}{x^3+8} = \lim_{x\to -2} \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x^2-2x+4)} = \lim_{x\to -2} \frac{1}{x^2-2x+4} = \frac{1}{12}$ BY DIRECT SUBST.

If you didn't remember the factorization for the sum of 2 cubes just use long division: or synthetic division

$$\begin{array}{r} x^2 - 2x + 4 \\ x+2 \,\overline{)\, x^3 \qquad\qquad +8} \\ -(x^3+2x^2) \\ \hline -2x^2 \\ -(-2x^2-4x) \\ \hline 4x+8 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$x^2-2x+4.$

$\boxed{2.3.20}$ $\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{(\cancel{8}+3\cdot 4h+3\cdot 2h^2+h^3)\cancel{-8}}{h}$

$= \lim_{h\to 0} \frac{12h+6h^2+h^3}{h}$ COMMON FACTOR OF h IN NUMERATOR.

$= \lim_{h\to 0} \frac{\cancel{h}(12+6h+h^2)}{\cancel{h}}$

$= \lim_{h\to 0} 12+6h+h^2$

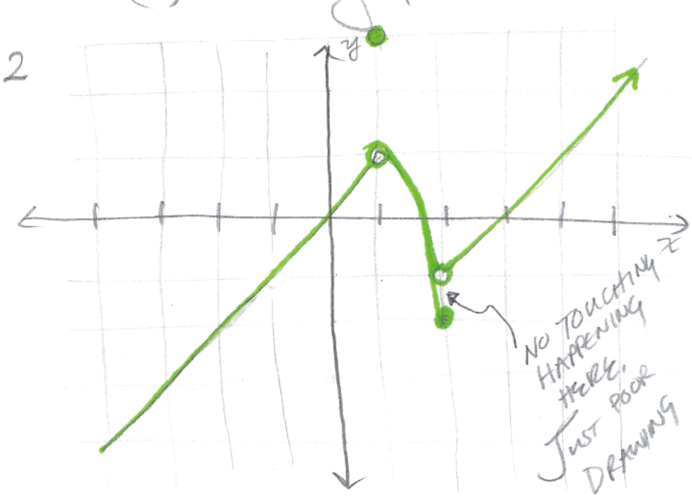
$= 12$ by direct substitution.

$\boxed{2.3.36}$ If $2x \le g(x) \le x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to 1} g(x)$.

We will apply the Sandwich Theorem. Since $\lim_{x\to 1} 2x = 2$ and $\lim_{x\to 1} x^4-x^2+2 = 2$ and these curves bound $g(x)$ above and below, the Sandwich Theorem requires $\lim_{x\to 1} g(x) = 2$ also.

$\boxed{2.3.48}$ Let $g(x) = \begin{cases} x & \text{if } x<1 \\ 3 & \text{if } x=1 \\ 2-x^2 & \text{if } 1<x\le 2 \\ x-3 & \text{if } x>2 \end{cases}$

(b) sketch graph



(a) (i) $\lim_{x\to 1^-} g(x) = 1$

(ii) $\lim_{x\to 1} g(x) = 1$

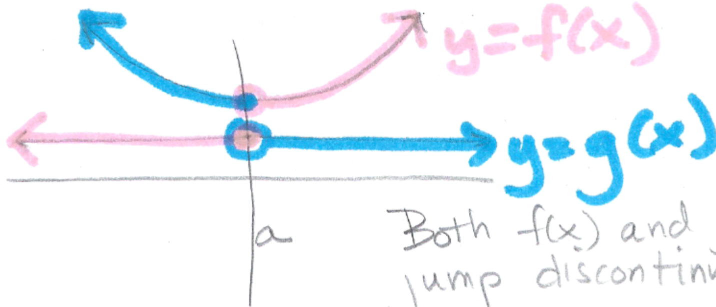
(iii) $g(1) = 3$

(iv) $\lim_{x\to 2^-} g(x) = -2$

(v) $\lim_{x\to 2^+} g(x) = 1$

(vi) $\lim_{x\to 2} g(x)$ D.N.E.

$\boxed{2.3.59}$ An example where $\lim_{x \to a} [f(x)g(x)]$ exists but neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ exists.



Both f(x) and g(x) have a jump discontinuity at x=a.
But when we multiply f(x)g(x) the result is a continuous function and $\lim_{x \to a} f(x)g(x) = f(a)g(a)$.