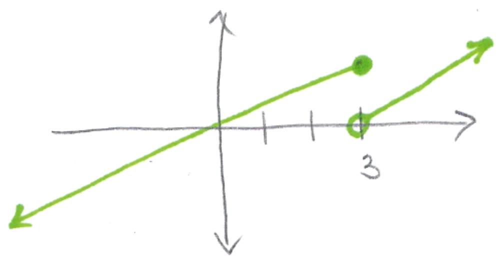
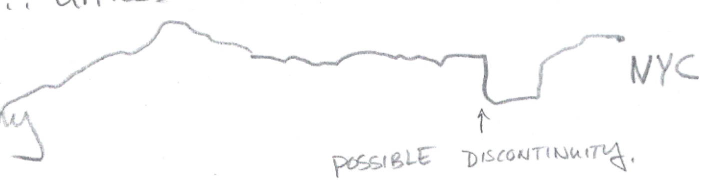
$\boxed{2.5.2}$ If f is continuous on $(-\infty, \infty)$, the changes in the function are gradual (no abrupt jumps), so the graph can indeed be "drawn" without lifting your pencil.

$\boxed{2.5.5}$ A function that is continuous everywhere except $x=3$ and is continuous from the left at 3.



There are many possible answers.
The important features are
① There is a jump discontinuity at $x=3$
② The value of the function at $x=3$ is contiguous with the left piece of the function.
③ Everything else about the function is "nice".

$\boxed{2.5.8}$ (a), (b), are continuous because the functions being described (temperature) change gradually
(d)(e) are discontinuous. Taxi fares are step functions based on particular fractions of a mile – plus waiting time. Current in a room is either "on" or "off" at a particular draw "
The change is not gradual.
(c) is likely to be continuous... unless there are sheer drops due west of NYC.
Most changes in topography are gradual.



$\boxed{2.5.19}$ Why is $f(x) = \begin{cases} \cos x & \text{if } x<0 \\ 0 & \text{if } x=0 \\ 1-x^2 & \text{if } x>0 \end{cases}$ discontinuous at $a=0$?

Notice $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \cos x = 1$
and $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} 1-x^2 = 1.$
$\Rightarrow$ So $\lim\limits_{x\to 0} f(x) = 1.$

But $f(0) = 0.$
Since $\lim\limits_{x\to 0} f(x) \neq f(0)$ the function is discontinuous.

$\boxed{2.5.23}$ $R(x) = x^2 + \sqrt{2x-1}$

If domain of $R(x)$ is all $\mathbb{R}$ s.t. $2x-1 \geq 0$. (alternatively all $x \geq \frac{1}{2}$.)

R is continuous on its domain because it is the sum of a polynomial (x^2) and the composition of continuous function $\sqrt{x}$ and $2x-1$.

$\boxed{2.5.25}$ $L(t) = e^{-5t}\cos(2\pi t)$

The domain of L is all $\mathbb{R}$.

Since $f(x) = e^x$ and $g(x) = \cos x$ are continuous functions so are the compositions $f(-5t)$ and $g(2\pi t)$.

Hence so is the product $f(-5t) \circ g(2\pi t) = L(t)$.

$\boxed{2.5.36}$ Show continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$$

Since $\sin x$ and $\cos x$ are continuous on all $\mathbb{R}$, we only need consider the behavior where the two pieces of these functions are joined — namely $x = \pi/4$.

$$\lim_{x \to \pi/4^-} f(x) = \lim_{x \to \frac{\pi}{4}^-} \sin x = \frac{\sqrt{2}}{2}$$

$$\lim_{x \to \frac{\pi}{4}^+} f(x) = \lim_{x \to \frac{\pi}{4}} \cos x = \frac{\sqrt{2}}{2}$$

So $\lim_{x \to \frac{\pi}{4}} f(x) = \frac{\sqrt{2}}{2} = f(\frac{\pi}{4})$. Hence f is continuous at $x = \pi/4$ (and consequintly at all real numbers.)

$\boxed{2.5.45}$ Since $f(x) = x^2 + 10\sin x$, it is a continuous function on $(-\infty, \infty)$.
Notice that $f(0) = 0$ and $f(100) \doteq 9997$.
Since $f(0) < 1000 < f(100)$ and f is continuous, the intermediate value theorem guarentees an input c with $0 < c < 100$ such that $f(c) = 1000$.

(Many of you correctly said that c was in the interval $(31, 32)$. While you are certainly correct, all the question asked was to verify c's existence. Not give an approximation for c.)

$\boxed{2.5.48}$ Show $\sqrt[3]{x} = 1 - x$ has a root in $(0,1)$.

Let $f(x) = \sqrt[3]{x} - 1 + x$. A root of the original equation will also be a root of $f(x)$ (i.e. a place where $f(x) = 0$.)
Since $f(x)$ is continuous everywhere and $f(0) = -1$ while $f(1) = 1$, the exist an input c in $(0,1)$ s.t. $f(c) = 0$. This value is also a root of $\sqrt[3]{x} = 1 - x$.

$\boxed{2.5.49}$ Show $\cos x = x$ has a root in $(0,1)$.

Consider $h(x) = \cos x - x$. Then h is continuous on $\mathbb{R}$.
$h(0) = 1$ and $h(1) \doteq -.46$. Since $1 > 0 > -.46$ the I.V.T guarentees c in $(0,1)$ so that $h(c) = 0$.
The solution will also satisfy $\cos x = x$ as desired.

$\boxed{2.5.65}$ Create 2 functions $D_1(t)$ and $D_2(t)$ that represent the Tibetan monk's altitude at time t on day 1 and 2 respectively. Assume that his altitude change is gradual (i.e. no teleporting to create discontinuities) - so $D_1(t)$ and $D_2(t)$ are continuous. Their difference $D_2(t) - D_1(t)$ is also continuous. At 7AM $D_2(7AM) - D_1(7AM)$ is POSITIVE (HE IS AT THE TOP AT THE BEGINNING OF DAY 2 AND THE BOTTOM AT THE BEGINNING OF DAY 1). AT 7PM, THE SITUATION IS REVERSED SO, $D_2(7PM) - D_1(7PM)$ IS NEGATIVE. AT SOME TIME OF THE DAY, THE DIFFERENCE WILL PASS THROUGH 0 BY THE I.V.T. THIS THE TIME WE ARE SEEKING WHEN THE MONK CROSSES THE SAME PT AT THE SAME TIME OF DAY.