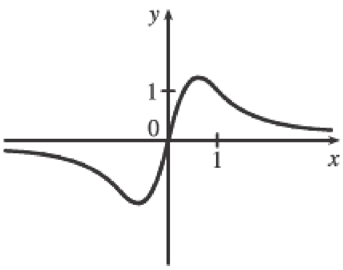
3. (a) $\lim_{x\to 2} f(x) = \infty$ (b) $\lim_{x\to -1^-} f(x) = \infty$ (c) $\lim_{x\to -1^+} f(x) = -\infty$

(d) $\lim_{x\to\infty} f(x) = 1$ (e) $\lim_{x\to -\infty} f(x) = 2$ (f) Vertical: $x = -1$, $x = 2$; Horizontal: $y = 1$, $y = 2$

5. $f(0) = 0, \quad f(1) = 1,$

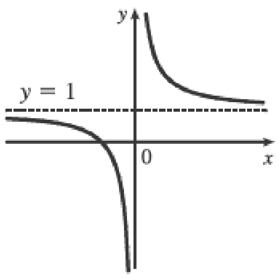
$\lim_{x\to\infty} f(x) = 0,$

f is odd



6. $\lim_{x\to 0^+} f(x) = \infty, \quad \lim_{x\to 0^-} f(x) = -\infty,$

$\lim_{x\to\infty} f(x) = 1, \quad \lim_{x\to -\infty} f(x) = 1$



15. $\lim_{x\to\infty} \dfrac{1}{2x+3} = \lim_{x\to\infty} \dfrac{1/x}{(2x+3)/x} = \dfrac{\lim_{x\to\infty}(1/x)}{\lim_{x\to\infty}(2+3/x)} = \dfrac{\lim_{x\to\infty}(1/x)}{\lim_{x\to\infty} 2 + 3\lim_{x\to\infty}(1/x)} = \dfrac{0}{2+3(0)} = \dfrac{0}{2} = 0$

19. Divide both the numerator and denominator by x^3 (the highest power of x that occurs in the denominator).

$$\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4} = \lim_{x\to\infty} \frac{\dfrac{x^3+5x}{x^3}}{\dfrac{2x^3-x^2+4}{x^3}} = \lim_{x\to\infty} \frac{1+\dfrac{5}{x^2}}{2-\dfrac{1}{x}+\dfrac{4}{x^3}} = \frac{\lim_{x\to\infty}\left(1+\dfrac{5}{x^2}\right)}{\lim_{x\to\infty}\left(2-\dfrac{1}{x}+\dfrac{4}{x^3}\right)}$$

$$= \frac{\lim_{x\to\infty} 1 + 5\lim_{x\to\infty}\dfrac{1}{x^2}}{\lim_{x\to\infty} 2 - \lim_{x\to\infty}\dfrac{1}{x} + 4\lim_{x\to\infty}\dfrac{1}{x^3}} = \frac{1+5(0)}{2-0+4(0)} = \frac{1}{2}$$

20. $\lim_{t\to -\infty} \dfrac{t^2+2}{t^3+t^2-1} = \lim_{t\to -\infty} \dfrac{(t^2+2)/t^3}{(t^3+t^2-1)/t^3} = \lim_{t\to -\infty} \dfrac{1/t+2/t^3}{1+1/t-1/t^3} = \dfrac{0+0}{1+0-0} = 0$

22. $\lim_{x\to\infty} \dfrac{x+2}{\sqrt{9x^2+1}} = \lim_{x\to\infty} \dfrac{(x+2)/x}{\sqrt{9x^2+1}/\sqrt{x^2}} = \lim_{x\to\infty} \dfrac{1+2/x}{\sqrt{9+1/x^2}} = \dfrac{1+0}{\sqrt{9+0}} = \dfrac{1}{3}$

28. $\lim_{x\to\infty} \cos x$ does not exist because as x increases $\cos x$ does not approach any one value, but oscillates between 1 and -1.

29. $\lim_{x\to\infty} \dfrac{x+x^3+x^5}{1-x^2+x^4} = \lim_{x\to\infty} \dfrac{(x+x^3+x^5)/x^4}{(1-x^2+x^4)/x^4}$ [divide by the highest power of x in the denominator]

$$= \lim_{x\to\infty} \frac{1/x^3+1/x+x}{1/x^4-1/x^2+1} = \infty$$

because $(1/x^3+1/x+x)\to\infty$ and $(1/x^4-1/x^2+1)\to 1$ as $x\to\infty$.

30. For $x>0$, $\sqrt{x^2+1} > \sqrt{x^2} = x$. So as $x\to\infty$, we have $\sqrt{x^2+1}\to\infty$, that is, $\lim_{x\to\infty}\sqrt{x^2+1} = \infty$.

33. $\lim_{x\to\infty} \dfrac{1-e^x}{1+2e^x} = \lim_{x\to\infty} \dfrac{(1-e^x)/e^x}{(1+2e^x)/e^x} = \lim_{x\to\infty} \dfrac{1/e^x-1}{1/e^x+2} = \dfrac{0-1}{0+2} = -\dfrac{1}{2}$