5. Using (1) with $f(x)=\dfrac{x-1}{x-2}$ and $P(3,2)$,

$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to 3}\frac{\dfrac{x-1}{x-2}-2}{x-3}=\lim_{x\to 3}\frac{\dfrac{x-1-2(x-2)}{x-2}}{x-3}$$

$$=\lim_{x\to 3}\frac{3-x}{(x-2)(x-3)}=\lim_{x\to 3}\frac{-1}{x-2}=\frac{-1}{1}=-1$$

Tangent line: $y-2=-1(x-3) \quad\Leftrightarrow\quad y-2=-x+3 \quad\Leftrightarrow\quad y=-x+5$

8. Using (1), $m=\lim\limits_{x\to 0}\dfrac{\dfrac{2x}{(x+1)^2}-0}{x-0}=\lim\limits_{x\to 0}\dfrac{2x}{x(x+1)^2}=\lim\limits_{x\to 0}\dfrac{2}{(x+1)^2}=\dfrac{2}{1^2}=2.$

Tangent line: $y-0=2(x-0) \quad\Leftrightarrow\quad y=2x$

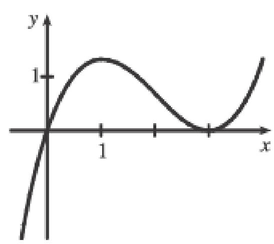
12. (a) **Runner A** runs the entire 100-meter race at the same velocity since the slope of the position function is constant. **Runner B** starts the race at a slower velocity than runner A, but finishes the race at a faster velocity.

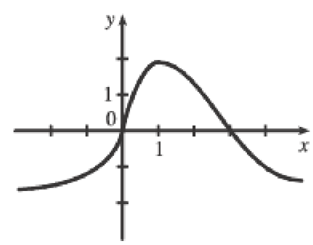
(b) The distance between the runners is the greatest at the time when the largest vertical line segment fits between the two graphs—this appears to be somewhere between 9 and 10 seconds.

(c) The runners had the same velocity when the slopes of their respective position functions are equal—this also appears to be at about 9.5 s. Note that the answers for parts (b) and (c) must be the same for these graphs because as soon as the velocity for runner B overtakes the velocity for runner A, the distance between the runners starts to decrease.

17. $g'(0)$ is the only negative value. The slope at $x=4$ is smaller than the slope at $x=2$ and both are smaller than the slope at $x=-2$. Thus, $g'(0)<0<g'(4)<g'(2)<g'(-2)$.

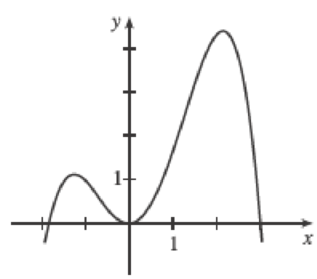
19. We begin by drawing a curve through the origin with a slope of 3 to satisfy $f(0)=0$ and $f'(0)=3$. Since $f'(1)=0$, we will round off our figure so that there is a horizontal tangent directly over $x=1$. Last, we make sure that the curve has a slope of -1 as we pass over $x=2$. Two of the many possibilities are shown.





20. We begin by drawing a curve through the origin with a slope of 0 to satisfy $g(0)=0$ and $g'(0)=0$. The curve should have a slope of -1, 3, and 1 as we pass over $x=-1$, 1, and 2, respectively.

Note: In the figure, $y'=0$ when $x\approx -1.27$ or 2.13.



25. Use Definition 2 with $f(x) = 3 - 2x + 4x^2$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{[3 - 2(a+h) + 4(a+h)^2] - (3 - 2a + 4a^2)}{h}$$

$$= \lim_{h\to 0} \frac{(3 - 2a - 2h + 4a^2 + 8ah + 4h^2) - (3 - 2a + 4a^2)}{h}$$

$$= \lim_{h\to 0} \frac{-2h + 8ah + 4h^2}{h} = \lim_{h\to 0} \frac{h(-2 + 8a + 4h)}{h} = \lim_{h\to 0} (-2 + 8a + 4h) = -2 + 8a$$

28. $$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{(a+h)^2 + 1}{(a+h) - 2} - \dfrac{a^2 + 1}{a - 2}}{h}$$

$$= \lim_{h\to 0} \frac{(a^2 + 2ah + h^2 + 1)(a - 2) - (a^2 + 1)(a + h - 2)}{h(a + h - 2)(a - 2)}$$

$$= \lim_{h\to 0} \frac{(a^3 - 2a^2 + 2a^2h - 4ah + ah^2 - 2h^2 + a - 2) - (a^3 + a^2h - 2a^2 + a + h - 2)}{h(a + h - 2)(a - 2)}$$

$$= \lim_{h\to 0} \frac{a^2h - 4ah + ah^2 - 2h^2 - h}{h(a + h - 2)(a - 2)} = \lim_{h\to 0} \frac{h(a^2 - 4a + ah - 2h - 1)}{h(a + h - 2)(a - 2)} = \lim_{h\to 0} \frac{a^2 - 4a + ah - 2h - 1}{(a + h - 2)(a - 2)}$$

$$= \frac{a^2 - 4a - 1}{(a - 2)^2}$$

31. By Definition 2, $\displaystyle\lim_{h\to 0} \frac{(1+h)^{10} - 1}{h} = f'(1)$, where $f(x) = x^{10}$ and $a = 1$.

Or: By Definition 2, $\displaystyle\lim_{h\to 0} \frac{(1+h)^{10} - 1}{h} = f'(0)$, where $f(x) = (1+x)^{10}$ and $a = 0$.

34. By Equation 3, $\displaystyle\lim_{x\to \pi/4} \frac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$ and $a = \pi/4$.