3. By the Product Rule, $f(x) = (x^3 + 2x)e^x \quad \Rightarrow$

$$f'(x) = (x^3 + 2x)(e^x)' + e^x(x^3 + 2x)' = (x^3 + 2x)e^x + e^x(3x^2 + 2)$$
$$= e^x[(x^3 + 2x) + (3x^2 + 2)] = e^x(x^3 + 3x^2 + 2x + 2)$$

4. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \quad \Rightarrow \quad g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x + 1)$.

5. By the Quotient Rule, $y = \dfrac{e^x}{x^2} \quad \Rightarrow \quad y' = \dfrac{x^2 \dfrac{d}{dx}(e^x) - e^x \dfrac{d}{dx}(x^2)}{(x^2)^2} = \dfrac{x^2(e^x) - e^x(2x)}{x^4} = \dfrac{xe^x(x-2)}{x^4} = \dfrac{e^x(x-2)}{x^3}$.

6. By the Quotient Rule, $y = \dfrac{e^x}{1+x} \quad \Rightarrow \quad y' = \dfrac{(1+x)e^x - e^x(1)}{(1+x)^2} = \dfrac{e^x + xe^x - e^x}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$.

7. $g(x) = \dfrac{3x-1}{2x+1} \quad \overset{\text{QR}}{\Rightarrow} \quad g'(x) = \dfrac{(2x+1)(3) - (3x-1)(2)}{(2x+1)^2} = \dfrac{6x + 3 - 6x + 2}{(2x+1)^2} = \dfrac{5}{(2x+1)^2}$

8. $f(t) = \dfrac{2t}{4+t^2} \quad \overset{\text{QR}}{\Rightarrow} \quad f'(t) = \dfrac{(4+t^2)(2) - (2t)(2t)}{(4+t^2)^2} = \dfrac{8 + 2t^2 - 4t^2}{(4+t^2)^2} = \dfrac{8 - 2t^2}{(4+t^2)^2}$

9. $V(x) = (2x^3 + 3)(x^4 - 2x) \quad \overset{\text{PR}}{\Rightarrow}$

$$V'(x) = (2x^3 + 3)(4x^3 - 2) + (x^4 - 2x)(6x^2) = (8x^6 + 8x^3 - 6) + (6x^6 - 12x^3) = 14x^6 - 4x^3 - 6$$

10. $Y(u) = (u^{-2} + u^{-3})(u^5 - 2u^2) \quad \overset{\text{PR}}{\Rightarrow}$

$$Y'(u) = (u^{-2} + u^{-3})(5u^4 - 4u) + (u^5 - 2u^2)(-2u^{-3} - 3u^{-4})$$
$$= (5u^2 - 4u^{-1} + 5u - 4u^{-2}) + (-2u^2 - 3u + 4u^{-1} + 6u^{-2}) = 3u^2 + 2u + 2u^{-2}$$

11. $F(y) = \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)(y + 5y^3) = (y^{-2} - 3y^{-4})(y + 5y^3) \quad \overset{\text{PR}}{\Rightarrow}$

$$F'(y) = (y^{-2} - 3y^{-4})(1 + 15y^2) + (y + 5y^3)(-2y^{-3} + 12y^{-5})$$
$$= (y^{-2} + 15 - 3y^{-4} - 45y^{-2}) + (-2y^{-2} + 12y^{-4} - 10 + 60y^{-2})$$
$$= 5 + 14y^{-2} + 9y^{-4} \text{ or } 5 + 14/y^2 + 9/y^4$$

12. $R(t) = (t + e^t)(3 - \sqrt{t}) \quad \overset{\text{PR}}{\Rightarrow}$

$$R'(t) = (t + e^t)\left(-\tfrac{1}{2}t^{-1/2}\right) + (3 - \sqrt{t})(1 + e^t)$$
$$= \left(-\tfrac{1}{2}t^{1/2} - \tfrac{1}{2}t^{-1/2}e^t\right) + \left(3 + 3e^t - \sqrt{t} - \sqrt{t}\,e^t\right) = 3 + 3e^t - \tfrac{3}{2}\sqrt{t} - \sqrt{t}\,e^t - e^t/(2\sqrt{t})$$

13. $y = \dfrac{x^3}{1-x^2} \quad \overset{\text{QR}}{\Rightarrow} \quad y' = \dfrac{(1-x^2)(3x^2) - x^3(-2x)}{(1-x^2)^2} = \dfrac{x^2(3 - 3x^2 + 2x^2)}{(1-x^2)^2} = \dfrac{x^2(3-x^2)}{(1-x^2)^2}$

14. $y = \dfrac{x+1}{x^3+x-2} \overset{\text{QR}}{\Rightarrow}$

$$y' = \frac{(x^3+x-2)(1)-(x+1)(3x^2+1)}{(x^3+x-2)^2} = \frac{x^3+x-2-3x^3-3x^2-x-1}{(x^3+x-2)^2} = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$$

or $-\dfrac{2x^3+3x^2+3}{(x-1)^2(x^2+x+2)^2}$

15. $y = \dfrac{t^2+2}{t^4-3t^2+1} \overset{\text{QR}}{\Rightarrow}$

$$y' = \frac{(t^4-3t^2+1)(2t)-(t^2+2)(4t^3-6t)}{(t^4-3t^2+1)^2} = \frac{2t[(t^4-3t^2+1)-(t^2+2)(2t^2-3)]}{(t^4-3t^2+1)^2}$$

$$= \frac{2t(t^4-3t^2+1-2t^4-4t^2+3t^2+6)}{(t^4-3t^2+1)^2} = \frac{2t(-t^4-4t^2+7)}{(t^4-3t^2+1)^2}$$

16. $y = \dfrac{t}{(t-1)^2} = \dfrac{t}{t^2-2t+1} \overset{\text{QR}}{\Rightarrow}$

$$y' = \frac{(t^2-2t+1)(1)-t(2t-2)}{[(t-1)^2]^2} = \frac{(t-1)^2-2t(t-1)}{(t-1)^4} = \frac{(t-1)[(t-1)-2t]}{(t-1)^4} = \frac{-t-1}{(t-1)^3}$$

17. $y = (r^2-2r)e^r \overset{\text{PR}}{\Rightarrow} y' = (r^2-2r)(e^r)+e^r(2r-2) = e^r(r^2-2r+2r-2) = e^r(r^2-2)$

18. $y = \dfrac{1}{s+ke^s} \overset{\text{QR}}{\Rightarrow} y' = \dfrac{(s+ke^s)(0)-(1)(1+ke^s)}{(s+ke^s)^2} = -\dfrac{1+ke^s}{(s+ke^s)^2}$

19. $y = \dfrac{v^3-2v\sqrt{v}}{v} = v^2-2\sqrt{v} = v^2-2v^{1/2} \Rightarrow y' = 2v-2\left(\frac{1}{2}\right)v^{-1/2} = 2v-v^{-1/2}$.

We can change the form of the answer as follows: $2v-v^{-1/2} = 2v-\dfrac{1}{\sqrt{v}} = \dfrac{2v\sqrt{v}-1}{\sqrt{v}} = \dfrac{2v^{3/2}-1}{\sqrt{v}}$

20. $z = w^{3/2}(w+ce^w) = w^{5/2}+cw^{3/2}e^w \Rightarrow z' = \frac{5}{2}w^{3/2}+c\left(w^{3/2}\cdot e^w+e^w\cdot\frac{3}{2}w^{1/2}\right) = \frac{5}{2}w^{3/2}+\frac{1}{2}cw^{1/2}e^w(2w+3)$

21. $f(t) = \dfrac{2t}{2+\sqrt{t}} \overset{\text{QR}}{\Rightarrow} f'(t) = \dfrac{(2+t^{1/2})(2)-2t\left(\frac{1}{2}t^{-1/2}\right)}{(2+\sqrt{t})^2} = \dfrac{4+2t^{1/2}-t^{1/2}}{(2+\sqrt{t})^2} = \dfrac{4+t^{1/2}}{(2+\sqrt{t})^2}$ or $\dfrac{4+\sqrt{t}}{(2+\sqrt{t})^2}$

22. $g(t) = \dfrac{t-\sqrt{t}}{t^{1/3}} = \dfrac{t}{t^{1/3}}-\dfrac{t^{1/2}}{t^{1/3}} = t^{2/3}-t^{1/6} \Rightarrow g'(t) = \frac{2}{3}t^{-1/3}-\frac{1}{6}t^{-5/6}$

23. $f(x) = \dfrac{A}{B+Ce^x} \overset{\text{QR}}{\Rightarrow} f'(x) = \dfrac{(B+Ce^x)\cdot 0-A(Ce^x)}{(B+Ce^x)^2} = -\dfrac{ACe^x}{(B+Ce^x)^2}$

24. $f(x) = \dfrac{1 - xe^x}{x + e^x} \overset{\text{QR}}{\Rightarrow} f'(x) = \dfrac{(x + e^x)(-xe^x)' - (1 - xe^x)(1 + e^x)}{(x + e^x)^2}$

$$\overset{\text{PR}}{\Rightarrow} f'(x) = \frac{(x + e^x)[-(xe^x + e^x \cdot 1)] - (1 + e^x - xe^x - xe^{2x})}{(x + e^x)^2}$$

$$= \frac{-x^2e^x - xe^x - xe^{2x} - e^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x + e^x)^2} = \frac{-x^2e^x - e^{2x} - e^x - 1}{(x + e^x)^2}$$

25. $f(x) = \dfrac{x}{x + c/x} \Rightarrow f'(x) = \dfrac{(x + c/x)(1) - x(1 - c/x^2)}{\left(x + \frac{c}{x}\right)^2} = \dfrac{x + c/x - x + c/x}{\left(\dfrac{x^2 + c}{x}\right)^2} = \dfrac{2c/x}{\dfrac{(x^2 + c)^2}{x^2}} \cdot \dfrac{x^2}{x^2} = \dfrac{2cx}{(x^2 + c)^2}$

26. $f(x) = \dfrac{ax + b}{cx + d} \Rightarrow f'(x) = \dfrac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} = \dfrac{acx + ad - acx - bc}{(cx + d)^2} = \dfrac{ad - bc}{(cx + d)^2}$

27. $f(x) = x^4e^x \Rightarrow f'(x) = x^4e^x + e^x \cdot 4x^3 = (x^4 + 4x^3)e^x \;\left[\text{or } x^3e^x(x + 4)\right] \Rightarrow$

$$f''(x) = (x^4 + 4x^3)e^x + e^x(4x^3 + 12x^2) = (x^4 + 4x^3 + 4x^3 + 12x^2)e^x$$

$$= (x^4 + 8x^3 + 12x^2)e^x \;\left[\text{or } x^2e^x(x + 2)(x + 6)\right]$$

30. $f(x) = \dfrac{x}{3 + e^x} \Rightarrow f'(x) = \dfrac{(3 + e^x)(1) - x(e^x)}{(3 + e^x)^2} = \dfrac{3 + e^x - xe^x}{(3 + e^x)^2} \Rightarrow$

$$f''(x) = \frac{(3 + e^x)^2\left[e^x - (xe^x + e^x \cdot 1)\right] - (3 + e^x - xe^x)(9 + 6e^x + e^x \cdot e^x)'}{\left[(3 + e^x)^2\right]^2}$$

$$= \frac{(3 + e^x)^2(-xe^x) - (3 + e^x - xe^x)(6e^x + e^x \cdot e^x + e^x \cdot e^x)}{(3 + e^x)^4}$$

$$= \frac{(3 + e^x)^2(-xe^x) - (3 + e^x - xe^x)(6e^x + 2e^{2x})}{(3 + e^x)^4} = \frac{(3 + e^x)^2(-xe^x) - (3 + e^x - xe^x)(2e^x)(3 + e^x)}{(3 + e^x)^4}$$

$$= \frac{(3 + e^x)e^x\left[(3 + e^x)(-x) - 2(3 + e^x - xe^x)\right]}{(3 + e^x)^4} = \frac{e^x(-3x - xe^x - 6 - 2e^x + 2xe^x)}{(3 + e^x)^3}$$

$$= \frac{e^x(xe^x - 2e^x - 3x - 6)}{(3 + e^x)^3}$$

43. We are given that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$.

(a) $(fg)'(5) = f(5)g'(5) + g(5)f'(5) = (1)(2) + (-3)(6) = 2 - 18 = -16$

(b) $\left(\dfrac{f}{g}\right)'(5) = \dfrac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \dfrac{(-3)(6) - (1)(2)}{(-3)^2} = -\dfrac{20}{9}$

(c) $\left(\dfrac{g}{f}\right)'(5) = \dfrac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2} = \dfrac{(1)(2) - (-3)(6)}{(1)^2} = 20$

44. We are given that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$.

(a) $h(x) = 5f(x) - 4g(x) \quad \Rightarrow \quad h'(x) = 5f'(x) - 4g'(x)$, so

$h'(2) = 5f'(2) - 4g'(2) = 5(-2) - 4(7) = -10 - 28 = -38$.

(b) $h(x) = f(x)g(x) \quad \Rightarrow \quad h'(x) = f(x)g'(x) + g(x)f'(x)$, so

$h'(2) = f(2)g'(2) + g(2)f'(2) = (-3)(7) + (4)(-2) = -21 - 8 = -29$.

(c) $h(x) = \dfrac{f(x)}{g(x)} \quad \Rightarrow \quad h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$, so

$$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{4(-2) - (-3)(7)}{4^2} = \frac{-8 + 21}{16} = \frac{13}{16}.$$

(d) $h(x) = \dfrac{g(x)}{1 + f(x)} \quad \Rightarrow \quad h'(x) = \dfrac{[1 + f(x)]\, g'(x) - g(x) f'(x)}{[1 + f(x)]^2}$, so

$$h'(2) = \frac{[1 + f(2)]\, g'(2) - g(2)\, f'(2)}{[1 + f(x)]^2} = \frac{[1 + (-3)](7) - 4(-2)}{[1 + (-3)]^2} = \frac{-14 + 8}{(-2)^2} = \frac{-6}{4} = -\frac{3}{2}.$$

45. $f(x) = e^x g(x) \quad \Rightarrow \quad f'(x) = e^x g'(x) + g(x)e^x = e^x[g'(x) + g(x)]$. $f'(0) = e^0[g'(0) + g(0)] = 1(5 + 2) = 7$

46. $$\frac{d}{dx}\left[\frac{h(x)}{x}\right] = \frac{xh'(x) - h(x) \cdot 1}{x^2} \quad \Rightarrow \quad \frac{d}{dx}\left[\frac{h(x)}{x}\right]_{x=2} = \frac{2h'(2) - h(2)}{2^2} = \frac{2(-3) - (4)}{4} = \frac{-10}{4} = -2.5$$

47. (a) From the graphs of f and g, we obtain the following values: $f(1) = 2$ since the point $(1, 2)$ is on the graph of f; $g(1) = 1$ since the point $(1, 1)$ is on the graph of g; $f'(1) = 2$ since the slope of the line segment between $(0, 0)$ and $(2, 4)$ is $\dfrac{4 - 0}{2 - 0} = 2$; $g'(1) = -1$ since the slope of the line segment between $(-2, 4)$ and $(2, 0)$ is $\dfrac{0 - 4}{2 - (-2)} = -1$.

Now $u(x) = f(x)g(x)$, so $u'(1) = f(1)g'(1) + g(1)\, f'(1) = 2 \cdot (-1) + 1 \cdot 2 = 0$.

(b) $v(x) = f(x)/g(x)$, so $v'(5) = \dfrac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \dfrac{2(-\frac{1}{3}) - 3 \cdot \frac{2}{3}}{2^2} = \dfrac{-\frac{8}{3}}{4} = -\dfrac{2}{3}$

58. (a) $$\frac{d}{dx}\left(\frac{1}{g(x)}\right) = \frac{g(x) \cdot \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2} \quad \text{[Quotient Rule]} \quad = \frac{g(x) \cdot 0 - 1 \cdot g'(x)}{[g(x)]^2} = \frac{0 - g'(x)}{[g(x)]^2} = -\frac{g'(x)}{[g(x)]^2}$$

(b) $y = \dfrac{1}{s + ke^s} \quad \Rightarrow \quad y' = -\dfrac{1 + ke^s}{(s + ke^s)^2}$

(c) $$\frac{d}{dx}\left(x^{-n}\right) = \frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{(x^n)'}{(x^n)^2} \quad \text{[by the Reciprocal Rule]} \quad = -\frac{nx^{n-1}}{x^{2n}} = -nx^{n-1-2n} = -nx^{-n-1}$$