29. $H(\theta) = \theta \sin \theta \quad \Rightarrow \quad H'(\theta) = \theta (\cos \theta) + (\sin \theta) \cdot 1 = \theta \cos \theta + \sin \theta \quad \Rightarrow$

$H''(\theta) = \theta (-\sin \theta) + (\cos \theta) \cdot 1 + \cos \theta = -\theta \sin \theta + 2 \cos \theta$

34. $y = \dfrac{\cos x}{2 + \sin x} \quad \Rightarrow \quad y' = \dfrac{(2 + \sin x)(-\sin x) - \cos x \cos x}{(2 + \sin x)^2} = \dfrac{-2 \sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2} = \dfrac{-2 \sin x - 1}{(2 + \sin x)^2} = 0$ when

$-2 \sin x - 1 = 0 \quad \Leftrightarrow \quad \sin x = -\frac{1}{2} \quad \Leftrightarrow \quad x = \frac{11\pi}{6} + 2\pi n$ or $x = \frac{7\pi}{6} + 2\pi n$, n an integer. So $y = \frac{1}{\sqrt{3}}$ or $y = -\frac{1}{\sqrt{3}}$ and

the points on the curve with horizontal tangents are: $\left(\frac{11\pi}{6} + 2\pi n, \frac{1}{\sqrt{3}} \right)$, $\left(\frac{7\pi}{6} + 2\pi n, -\frac{1}{\sqrt{3}} \right)$, n an integer.

39. $\displaystyle\lim_{x \to 0} \dfrac{\sin 3x}{x} = \lim_{x \to 0} \dfrac{3 \sin 3x}{3x}$ [multiply numerator and denominator by 3]

$\qquad\qquad = 3 \displaystyle\lim_{3x \to 0} \dfrac{\sin 3x}{3x}$ [as $x \to 0$, $3x \to 0$]

$\qquad\qquad = 3 \displaystyle\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta}$ [let $\theta = 3x$]

$\qquad\qquad = 3(1)$ [Equation 2]

$\qquad\qquad = 3$

40. $\displaystyle\lim_{x \to 0} \dfrac{\sin 4x}{\sin 6x} = \lim_{x \to 0} \left(\dfrac{\sin 4x}{x} \cdot \dfrac{x}{\sin 6x} \right) = \lim_{x \to 0} \dfrac{4 \sin 4x}{4x} \cdot \lim_{x \to 0} \dfrac{6x}{6 \sin 6x} = 4 \lim_{x \to 0} \dfrac{\sin 4x}{4x} \cdot \dfrac{1}{6} \lim_{x \to 0} \dfrac{6x}{\sin 6x} = 4(1) \cdot \dfrac{1}{6}(1) = \dfrac{2}{3}$

41. $\displaystyle\lim_{t \to 0} \dfrac{\tan 6t}{\sin 2t} = \lim_{t \to 0} \left(\dfrac{\sin 6t}{t} \cdot \dfrac{1}{\cos 6t} \cdot \dfrac{t}{\sin 2t} \right) = \lim_{t \to 0} \dfrac{6 \sin 6t}{6t} \cdot \lim_{t \to 0} \dfrac{1}{\cos 6t} \cdot \lim_{t \to 0} \dfrac{2t}{2 \sin 2t}$

$\qquad\qquad = 6 \displaystyle\lim_{t \to 0} \dfrac{\sin 6t}{6t} \cdot \lim_{t \to 0} \dfrac{1}{\cos 6t} \cdot \dfrac{1}{2} \lim_{t \to 0} \dfrac{2t}{\sin 2t} = 6(1) \cdot \dfrac{1}{1} \cdot \dfrac{1}{2}(1) = 3$

42. $\displaystyle\lim_{\theta \to 0} \dfrac{\cos \theta - 1}{\sin \theta} = \lim_{\theta \to 0} \dfrac{\dfrac{\cos \theta - 1}{\theta}}{\dfrac{\sin \theta}{\theta}} = \dfrac{\displaystyle\lim_{\theta \to 0} \dfrac{\cos \theta - 1}{\theta}}{\displaystyle\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta}} = \dfrac{0}{1} = 0$

43. $\displaystyle\lim_{\theta \to 0} \dfrac{\sin(\cos \theta)}{\sec \theta} = \dfrac{\sin\left(\displaystyle\lim_{\theta \to 0} \cos \theta \right)}{\displaystyle\lim_{\theta \to 0} \sec \theta} = \dfrac{\sin 1}{1} = \sin 1$

44. $\displaystyle\lim_{t \to 0} \dfrac{\sin^2 3t}{t^2} = \lim_{t \to 0} \left(\dfrac{\sin 3t}{t} \cdot \dfrac{\sin 3t}{t} \right) = \lim_{t \to 0} \dfrac{\sin 3t}{t} \cdot \lim_{t \to 0} \dfrac{\sin 3t}{t} = \left(\lim_{t \to 0} \dfrac{\sin 3t}{t} \right)^2 = \left(3 \lim_{t \to 0} \dfrac{\sin 3t}{3t} \right)^2 = (3 \cdot 1)^2 = 9$

45. Divide numerator and denominator by θ.   ($\sin \theta$ also works.)

$\displaystyle\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta + \tan \theta} = \lim_{\theta \to 0} \dfrac{\dfrac{\sin \theta}{\theta}}{1 + \dfrac{\sin \theta}{\theta} \cdot \dfrac{1}{\cos \theta}} = \dfrac{\displaystyle\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta}}{1 + \displaystyle\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta} \cdot \lim_{\theta \to 0} \dfrac{1}{\cos \theta}} = \dfrac{1}{1 + 1 \cdot 1} = \dfrac{1}{2}$

46. $\displaystyle\lim_{x \to 0} \dfrac{\sin(x^2)}{x} = \lim_{x \to 0} \left[x \cdot \dfrac{\sin(x^2)}{x \cdot x} \right] = \lim_{x \to 0} x \cdot \lim_{x \to 0} \dfrac{\sin(x^2)}{x^2} = 0 \cdot \lim_{y \to 0+} \dfrac{\sin y}{y}$ $\left[\text{where } y = x^2 \right] = 0 \cdot 1 = 0$

47. $\displaystyle\lim_{x\to\pi/4}\frac{1-\tan x}{\sin x-\cos x}=\lim_{x\to\pi/4}\frac{\left(1-\dfrac{\sin x}{\cos x}\right)\cdot\cos x}{(\sin x-\cos x)\cdot\cos x}=\lim_{x\to\pi/4}\frac{\cos x-\sin x}{(\sin x-\cos x)\cos x}=\lim_{x\to\pi/4}\frac{-1}{\cos x}=\frac{-1}{1/\sqrt{2}}=-\sqrt{2}$

48. $\displaystyle\lim_{x\to1}\frac{\sin(x-1)}{x^2+x-2}=\lim_{x\to1}\frac{\sin(x-1)}{(x+2)(x-1)}=\lim_{x\to1}\frac{1}{x+2}\lim_{x\to1}\frac{\sin(x-1)}{x-1}=\frac{1}{3}\cdot1=\frac{1}{3}$