7. $F(x) = (x^4 + 3x^2 - 2)^5 \quad \Rightarrow \quad F'(x) = 5(x^4 + 3x^2 - 2)^4 \cdot \frac{d}{dx}\left(x^4 + 3x^2 - 2\right) = 5(x^4 + 3x^2 - 2)^4(4x^3 + 6x)$

$\left[\text{or } 10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)\right]$

8. $F(x) = (4x - x^2)^{100} \quad \Rightarrow \quad F'(x) = 100(4x - x^2)^{99} \cdot \frac{d}{dx}(4x - x^2) = 100(4x - x^2)^{99}(4 - 2x)$

$\left[\text{or } 200x^{99}(x - 2)(x - 4)^{99}\right]$

9. $F(x) = \sqrt[4]{1 + 2x + x^3} = (1 + 2x + x^3)^{1/4} \quad \Rightarrow$

$$F'(x) = \tfrac{1}{4}(1 + 2x + x^3)^{-3/4} \cdot \frac{d}{dx}\left(1 + 2x + x^3\right) = \frac{1}{4(1 + 2x + x^3)^{3/4}} \cdot (2 + 3x^2) = \frac{2 + 3x^2}{4(1 + 2x + x^3)^{3/4}}$$

$$= \frac{2 + 3x^2}{4\sqrt[4]{(1 + 2x + x^3)^3}}$$

10. $f(x) = (1 + x^4)^{2/3} \quad \Rightarrow \quad f'(x) = \tfrac{2}{3}(1 + x^4)^{-1/3}(4x^3) = \dfrac{8x^3}{3\sqrt[3]{1 + x^4}}$

11. $g(t) = \dfrac{1}{(t^4 + 1)^3} = (t^4 + 1)^{-3} \quad \Rightarrow \quad g'(t) = -3(t^4 + 1)^{-4}(4t^3) = -12t^3(t^4 + 1)^{-4} = \dfrac{-12t^3}{(t^4 + 1)^4}$

12. $f(t) = \sqrt[3]{1 + \tan t} = (1 + \tan t)^{1/3} \quad \Rightarrow \quad f'(t) = \tfrac{1}{3}(1 + \tan t)^{-2/3}\sec^2 t = \dfrac{\sec^2 t}{3\sqrt[3]{(1 + \tan t)^2}}$

13. $y = \cos(a^3 + x^3) \quad \Rightarrow \quad y' = -\sin(a^3 + x^3) \cdot 3x^2 \quad [a^3 \text{ is just a constant}] \quad = -3x^2 \sin(a^3 + x^3)$

14. $y = a^3 + \cos^3 x \quad \Rightarrow \quad y' = 3(\cos x)^2(-\sin x) \quad [a^3 \text{ is just a constant}] \quad = -3 \sin x \cos^2 x$

15. $y = xe^{-kx} \quad \Rightarrow \quad y' = x\left[e^{-kx}(-k)\right] + e^{-kx} \cdot 1 = e^{-kx}(-kx + 1) \quad \left[\text{or } (1 - kx)e^{-kx}\right]$

16. $y = 3\cot(n\theta) \quad \Rightarrow \quad y' = 3[-\csc^2(n\theta) \cdot n] = -3n\csc^2(n\theta)$

17. $g(x) = (1 + 4x)^5(3 + x - x^2)^8 \quad \Rightarrow$

$$g'(x) = (1 + 4x)^5 \cdot 8(3 + x - x^2)^7(1 - 2x) + (3 + x - x^2)^8 \cdot 5(1 + 4x)^4 \cdot 4$$

$$= 4(1 + 4x)^4(3 + x - x^2)^7\left[2(1 + 4x)(1 - 2x) + 5(3 + x - x^2)\right]$$

$$= 4(1 + 4x)^4(3 + x - x^2)^7\left[(2 + 4x - 16x^2) + (15 + 5x - 5x^2)\right] = 4(1 + 4x)^4(3 + x - x^2)^7(17 + 9x - 21x^2)$$

18. $h(t) = (t^4 - 1)^3(t^3 + 1)^4 \quad \Rightarrow$

$$h'(t) = (t^4 - 1)^3 \cdot 4(t^3 + 1)^3(3t^2) + (t^3 + 1)^4 \cdot 3(t^4 - 1)^2(4t^3)$$

$$= 12t^2(t^4 - 1)^2(t^3 + 1)^3\left[(t^4 - 1) + t(t^3 + 1)\right] = 12t^2(t^4 - 1)^2(t^3 + 1)^3(2t^4 + t - 1)$$

19. $y = (2x-5)^4(8x^2-5)^{-3} \quad \Rightarrow$

$$\begin{aligned} y' &= 4(2x-5)^3(2)(8x^2-5)^{-3} + (2x-5)^4(-3)(8x^2-5)^{-4}(16x) \\ &= 8(2x-5)^3(8x^2-5)^{-3} - 48x(2x-5)^4(8x^2-5)^{-4} \end{aligned}$$

[This simplifies to $8(2x-5)^3(8x^2-5)^{-4}(-4x^2+30x-5)$.]

20. $y = (x^2+1)\left(x^2+2\right)^{1/3} \quad \Rightarrow$

$$y' = 2x\left(x^2+2\right)^{1/3} + \left(x^2+1\right)\left(\tfrac{1}{3}\right)\left(x^2+2\right)^{-2/3}(2x) = 2x\left(x^2+2\right)^{1/3}\left[1 + \frac{x^2+1}{3(x^2+2)}\right]$$

21. $y = \left(\dfrac{x^2+1}{x^2-1}\right)^3 \quad \Rightarrow$

$$\begin{aligned} y' &= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2} \\ &= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{2x[x^2-1-(x^2+1)]}{(x^2-1)^2} = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{2x(-2)}{(x^2-1)^2} = \frac{-12x(x^2+1)^2}{(x^2-1)^4} \end{aligned}$$

22. $y = e^{-5x}\cos 3x \quad \Rightarrow \quad y' = e^{-5x}(-3\sin 3x) + (\cos 3x)(-5e^{-5x}) = -e^{-5x}(3\sin 3x + 5\cos 3x)$

23. $y = e^{x\cos x} \quad \Rightarrow \quad y' = e^{x\cos x} \cdot \dfrac{d}{dx}(x\cos x) = e^{x\cos x}\left[x(-\sin x) + (\cos x)\cdot 1\right] = e^{x\cos x}(\cos x - x\sin x)$

24. Using Formula 5 and the Chain Rule, $y = 10^{1-x^2} \quad \Rightarrow \quad y' = 10^{1-x^2}(\ln 10)\cdot\dfrac{d}{dx}\left(1-x^2\right) = -2x(\ln 10)10^{1-x^2}$.

25. $F(z) = \sqrt{\dfrac{z-1}{z+1}} = \left(\dfrac{z-1}{z+1}\right)^{1/2} \quad \Rightarrow$

$$\begin{aligned} F'(z) &= \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2} \cdot \frac{d}{dz}\left(\frac{z-1}{z+1}\right) = \frac{1}{2}\left(\frac{z+1}{z-1}\right)^{1/2} \cdot \frac{(z+1)(1)-(z-1)(1)}{(z+1)^2} \\ &= \frac{1}{2}\frac{(z+1)^{1/2}}{(z-1)^{1/2}} \cdot \frac{z+1-z+1}{(z+1)^2} = \frac{1}{2}\frac{(z+1)^{1/2}}{(z-1)^{1/2}} \cdot \frac{2}{(z+1)^2} = \frac{1}{(z-1)^{1/2}(z+1)^{3/2}} \end{aligned}$$

26. $G(y) = \dfrac{(y-1)^4}{(y^2+2y)^5} \quad \Rightarrow$

$$\begin{aligned} G'(y) &= \frac{(y^2+2y)^5 \cdot 4(y-1)^3 \cdot 1 - (y-1)^4 \cdot 5(y^2+2y)^4(2y+2)}{\left[(y^2+2y)^5\right]^2} \\ &= \frac{2(y^2+2y)^4(y-1)^3\left[2(y^2+2y) - 5(y-1)(y+1)\right]}{(y^2+2y)^{10}} \\ &= \frac{2(y-1)^3\left[(2y^2+4y)+(-5y^2+5)\right]}{(y^2+2y)^6} = \frac{2(y-1)^3(-3y^2+4y+5)}{(y^2+2y)^6} \end{aligned}$$

27. $y = \dfrac{r}{\sqrt{r^2+1}} \quad \Rightarrow$

$$y' = \frac{\sqrt{r^2+1}\,(1) - r\cdot\frac{1}{2}(r^2+1)^{-1/2}(2r)}{\left(\sqrt{r^2+1}\right)^2} = \frac{\sqrt{r^2+1} - \dfrac{r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2} = \frac{\dfrac{\sqrt{r^2+1}\,\sqrt{r^2+1} - r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2}$$

$$= \frac{(r^2+1) - r^2}{\left(\sqrt{r^2+1}\right)^3} = \frac{1}{(r^2+1)^{3/2}} \text{ or } (r^2+1)^{-3/2}$$

Another solution: Write y as a product and make use of the Product Rule. $y = r(r^2+1)^{-1/2} \quad \Rightarrow$

$y' = r\cdot -\frac{1}{2}(r^2+1)^{-3/2}(2r) + (r^2+1)^{-1/2}\cdot 1 = (r^2+1)^{-3/2}[-r^2 + (r^2+1)^1] = (r^2+1)^{-3/2}(1) = (r^2+1)^{-3/2}$.

The step that students usually have trouble with is factoring out $(r^2+1)^{-3/2}$. But this is no different than factoring out x^2 from x^2+x^5; that is, we are just factoring out a factor with the *smallest* exponent that appears on it. In this case, $-\frac{3}{2}$ is smaller than $-\frac{1}{2}$.

28. $y = \dfrac{e^u - e^{-u}}{e^u + e^{-u}} \quad \Rightarrow$

$$y' = \frac{(e^u+e^{-u})(e^u-(-e^u)) - (e^u-e^{-u})(e^u+(-e^{-u}))}{(e^u+e^{-u})^2} = \frac{e^{2u}+e^0+e^0+e^{-2u} - (e^{2u}-e^0-e^0+e^{-2u})}{(e^u+e^{-u})^2}$$

$$= \frac{4e^0}{(e^u+e^{-u})^2} = \frac{4}{(e^u+e^{-u})^2}$$

29. $y = \sin(\tan 2x) \quad \Rightarrow \quad y' = \cos(\tan 2x)\cdot\dfrac{d}{dx}(\tan 2x) = \cos(\tan 2x)\cdot\sec^2(2x)\cdot\dfrac{d}{dx}(2x) = 2\cos(\tan 2x)\sec^2(2x)$

30. $G(y) = \left(\dfrac{y^2}{y+1}\right)^5 \quad \Rightarrow \quad G'(y) = 5\left(\dfrac{y^2}{y+1}\right)^4\cdot\dfrac{(y+1)(2y) - y^2(1)}{(y+1)^2} = 5\cdot\dfrac{y^8}{(y+1)^4}\cdot\dfrac{y(2y+2-y)}{(y+1)^2} = \dfrac{5y^9(y+2)}{(y+1)^6}$

31. Using Formula 5 and the Chain Rule, $y = 2^{\sin \pi x} \quad \Rightarrow$

$y' = 2^{\sin\pi x}(\ln 2)\cdot\dfrac{d}{dx}(\sin\pi x) = 2^{\sin\pi x}(\ln 2)\cdot\cos\pi x\cdot\pi = 2^{\sin\pi x}(\pi\ln 2)\cos\pi x$

32. $y = \tan^2(3\theta) = (\tan 3\theta)^2 \quad \Rightarrow \quad y' = 2(\tan 3\theta)\cdot\dfrac{d}{d\theta}(\tan 3\theta) = 2\tan 3\theta\cdot\sec^2 3\theta\cdot 3 = 6\tan 3\theta\sec^2 3\theta$

33. $y = \sec^2 x + \tan^2 x = (\sec x)^2 + (\tan x)^2 \quad \Rightarrow$

$y' = 2(\sec x)(\sec x\,\tan x) + 2(\tan x)(\sec^2 x) = 2\sec^2 x\,\tan x + 2\sec^2 x\,\tan x = 4\sec^2 x\,\tan x$

34. $y = x\sin\dfrac{1}{x} \quad \Rightarrow \quad y' = \sin\dfrac{1}{x} + x\cos\dfrac{1}{x}\left(-\dfrac{1}{x^2}\right) = \sin\dfrac{1}{x} - \dfrac{1}{x}\cos\dfrac{1}{x}$

35. $y = \cos\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right) \Rightarrow$

$$y' = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{d}{dx}\left(\frac{1-e^{2x}}{1+e^{2x}}\right) = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{(1+e^{2x})(-2e^{2x})-(1-e^{2x})(2e^{2x})}{(1+e^{2x})^2}$$

$$= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{-2e^{2x}\left[(1+e^{2x})+(1-e^{2x})\right]}{(1+e^{2x})^2} = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\cdot\frac{-2e^{2x}(2)}{(1+e^{2x})^2} = \frac{4e^{2x}}{(1+e^{2x})^2}\cdot\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$$

36. $f(t) = \sqrt{\dfrac{t}{t^2+4}} = \left(\dfrac{t}{t^2+4}\right)^{1/2} \Rightarrow$

$$f'(t) = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-1/2}\cdot\frac{d}{dt}\left(\frac{t}{t^2+4}\right) = \frac{1}{2}\left(\frac{t^2+4}{t}\right)^{1/2}\cdot\frac{(t^2+4)(1)-t(2t)}{(t^2+4)^2}$$

$$= \frac{(t^2+4)^{1/2}}{2t^{1/2}}\cdot\frac{t^2+4-2t^2}{(t^2+4)^2} = \frac{4-t^2}{2t^{1/2}(t^2+4)^{3/2}}$$

37. $y = \cot^2(\sin\theta) = [\cot(\sin\theta)]^2 \Rightarrow$

$$y' = 2[\cot(\sin\theta)]\cdot\frac{d}{d\theta}[\cot(\sin\theta)] = 2\cot(\sin\theta)\cdot[-\csc^2(\sin\theta)\cdot\cos\theta] = -2\cos\theta\,\cot(\sin\theta)\,\csc^2(\sin\theta)$$

38. $y = e^{k\tan\sqrt{x}} \Rightarrow y' = e^{k\tan\sqrt{x}}\cdot\dfrac{d}{dx}\left(k\tan\sqrt{x}\right) = e^{k\tan\sqrt{x}}\left(k\sec^2\sqrt{x}\cdot\tfrac{1}{2}x^{-1/2}\right) = \dfrac{k\sec^2\sqrt{x}}{2\sqrt{x}}\,e^{k\tan\sqrt{x}}$

39. $f(t) = \tan(e^t) + e^{\tan t} \Rightarrow f'(t) = \sec^2(e^t)\cdot\dfrac{d}{dt}(e^t) + e^{\tan t}\cdot\dfrac{d}{dt}(\tan t) = \sec^2(e^t)\cdot e^t + e^{\tan t}\cdot\sec^2 t$

40. $y = \sin(\sin(\sin x)) \Rightarrow y' = \cos(\sin(\sin x))\dfrac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x))\,\cos(\sin x)\,\cos x$

41. $f(t) = \sin^2\left(e^{\sin^2 t}\right) = \left[\sin\left(e^{\sin^2 t}\right)\right]^2 \Rightarrow$

$$f'(t) = 2\left[\sin\left(e^{\sin^2 t}\right)\right]\cdot\frac{d}{dt}\sin\left(e^{\sin^2 t}\right) = 2\sin\left(e^{\sin^2 t}\right)\cdot\cos\left(e^{\sin^2 t}\right)\cdot\frac{d}{dt}e^{\sin^2 t}$$

$$= 2\sin\left(e^{\sin^2 t}\right)\cos\left(e^{\sin^2 t}\right)\cdot e^{\sin^2 t}\cdot\frac{d}{dt}\sin^2 t = 2\sin\left(e^{\sin^2 t}\right)\cos\left(e^{\sin^2 t}\right)e^{\sin^2 t}\cdot 2\sin t\,\cos t$$

$$= 4\sin\left(e^{\sin^2 t}\right)\cos\left(e^{\sin^2 t}\right)e^{\sin^2 t}\sin t\,\cos t$$

42. $y = \sqrt{x+\sqrt{x+\sqrt{x}}} \Rightarrow y' = \frac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-1/2}\left[1+\frac{1}{2}\left(x+\sqrt{x}\right)^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right]$

43. $g(x) = (2ra^{rx}+n)^p \Rightarrow$

$$g'(x) = p(2ra^{rx}+n)^{p-1}\cdot\frac{d}{dx}(2ra^{rx}+n) = p(2ra^{rx}+n)^{p-1}\cdot 2ra^{rx}(\ln a)\cdot r = 2r^2p(\ln a)(2ra^{rx}+n)^{p-1}a^{rx}$$

44. $y = 2^{3^{x^2}} \quad \Rightarrow \quad y' = 2^{3^{x^2}}(\ln 2)\frac{d}{dx}\left(3^{x^2}\right) = 2^{3^{x^2}}(\ln 2)3^{x^2}(\ln 3)(2x)$

45. $y = \cos\sqrt{\sin(\tan \pi x)} = \cos(\sin(\tan \pi x))^{1/2} \quad \Rightarrow$

$$y' = -\sin(\sin(\tan \pi x))^{1/2} \cdot \frac{d}{dx}(\sin(\tan \pi x))^{1/2} = -\sin(\sin(\tan \pi x))^{1/2} \cdot \tfrac{1}{2}(\sin(\tan \pi x))^{-1/2} \cdot \frac{d}{dx}(\sin(\tan \pi x))$$

$$= \frac{-\sin\sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}} \cdot \cos(\tan \pi x) \cdot \frac{d}{dx}\tan \pi x = \frac{-\sin\sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}} \cdot \cos(\tan \pi x) \cdot \sec^2(\pi x) \cdot \pi$$

$$= \frac{-\pi \cos(\tan \pi x)\sec^2(\pi x)\sin\sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}}$$

46. $y = \left[x + (x + \sin^2 x)^3\right]^4 \quad \Rightarrow \quad y' = 4\left[x + (x + \sin^2 x)^3\right]^3 \cdot \left[1 + 3(x + \sin^2 x)^2 \cdot (1 + 2\sin x \, \cos x)\right]$

47. $h(x) = \sqrt{x^2 + 1} \quad \Rightarrow \quad h'(x) = \dfrac{1}{2}(x^2 + 1)^{-1/2}(2x) = \dfrac{x}{\sqrt{x^2 + 1}} \quad \Rightarrow$

$$h''(x) = \frac{\sqrt{x^2+1} \cdot 1 - x\left[\frac{1}{2}(x^2+1)^{-1/2}(2x)\right]}{\left(\sqrt{x^2+1}\right)^2} = \frac{(x^2+1)^{-1/2}\left[(x^2+1) - x^2\right]}{(x^2+1)^1} = \frac{1}{(x^2+1)^{3/2}}$$

50. $y = e^{e^x} \quad \Rightarrow \quad y' = e^{e^x} \cdot (e^x)' = e^{e^x} \cdot e^x \quad \Rightarrow$

$$y'' = e^{e^x} \cdot (e^x)' + e^x \cdot \left(e^{e^x}\right)' = e^{e^x} \cdot e^x + e^x \cdot e^{e^x} \cdot e^x = e^{e^x} \cdot e^x(1 + e^x) \quad \text{or} \quad e^{e^x + x}(1 + e^x)$$

59. For the tangent line to be horizontal, $f'(x) = 0$. $f(x) = 2\sin x + \sin^2 x \quad \Rightarrow \quad f'(x) = 2\cos x + 2\sin x \, \cos x = 0 \quad \Leftrightarrow$ $2\cos x(1 + \sin x) = 0 \quad \Leftrightarrow \quad \cos x = 0$ or $\sin x = -1$, so $x = \frac{\pi}{2} + 2n\pi$ or $\frac{3\pi}{2} + 2n\pi$, where n is any integer. Now $f\left(\frac{\pi}{2}\right) = 3$ and $f\left(\frac{3\pi}{2}\right) = -1$, so the points on the curve with a horizontal tangent are $\left(\frac{\pi}{2} + 2n\pi, 3\right)$ and $\left(\frac{3\pi}{2} + 2n\pi, -1\right)$, where n is any integer.

63. (a) $h(x) = f(g(x)) \quad \Rightarrow \quad h'(x) = f'(g(x)) \cdot g'(x)$, so $h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 5 \cdot 6 = 30$.

(b) $H(x) = g(f(x)) \quad \Rightarrow \quad H'(x) = g'(f(x)) \cdot f'(x)$, so $H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 9 \cdot 4 = 36$.

66. (a) $h(x) = f(f(x)) \quad \Rightarrow \quad h'(x) = f'(f(x))f'(x)$. So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

(b) $g(x) = f(x^2) \quad \Rightarrow \quad g'(x) = f'(x^2) \cdot \dfrac{d}{dx}(x^2) = f'(x^2)(2x)$. So $g'(2) = f'(2^2)(2 \cdot 2) = 4f'(4) \approx 4(2) = 8$.