1. (a) $\frac{d}{dx}\left(xy + 2x + 3x^2\right) = \frac{d}{dx}(4) \;\Rightarrow\; (x \cdot y' + y \cdot 1) + 2 + 6x = 0 \;\Rightarrow\; xy' = -y - 2 - 6x \;\Rightarrow$
$y' = \frac{-y - 2 - 6x}{x}$ or $y' = -6 - \frac{y+2}{x}$.

(b) $xy + 2x + 3x^2 = 4 \;\Rightarrow\; xy = 4 - 2x - 3x^2 \;\Rightarrow\; y = \frac{4 - 2x - 3x^2}{x} = \frac{4}{x} - 2 - 3x$, so $y' = -\frac{4}{x^2} - 3$.

(c) From part (a), $y' = \frac{-y - 2 - 6x}{x} = \frac{-(4/x - 2 - 3x) - 2 - 6x}{x} = \frac{-4/x - 3x}{x} = -\frac{4}{x^2} - 3$.

4. (a) $\frac{d}{dx}\left(\cos x + \sqrt{y}\right) = \frac{d}{dx}(5) \;\Rightarrow\; -\sin x + \frac{1}{2}y^{-1/2} \cdot y' = 0 \;\Rightarrow\; \frac{1}{2\sqrt{y}} \cdot y' = \sin x \;\Rightarrow\; y' = 2\sqrt{y}\sin x$

(b) $\cos x + \sqrt{y} = 5 \;\Rightarrow\; \sqrt{y} = 5 - \cos x \;\Rightarrow\; y = (5 - \cos x)^2$, so $y' = 2(5 - \cos x)'(\sin x) = 2\sin x(5 - \cos x)$.

(c) From part (a), $y' = 2\sqrt{y}\sin x = 2\sqrt{(5 - \cos x)^2} = 2(5 - \cos x)\sin x \quad [\text{since } 5 - \cos x > 0]$.

12. $\frac{d}{dx}(1 + x) = \frac{d}{dx}\left[\sin(xy^2)\right] \;\Rightarrow\; 1 = [\cos(xy^2)](x \cdot 2y\,y' + y^2 \cdot 1) \;\Rightarrow\; 1 = 2xy\cos(xy^2)\,y' + y^2\cos(xy^2) \;\Rightarrow$

$1 - y^2\cos(xy^2) = 2xy\cos(xy^2)\,y' \;\Rightarrow\; y' = \frac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}$

13. $\frac{d}{dx}(4\cos x\,\sin y) = \frac{d}{dx}(1) \;\Rightarrow\; 4\left[\cos x \cdot \cos y \cdot y' + \sin y \cdot (-\sin x)\right] = 0 \;\Rightarrow$

$y'(4\cos x\,\cos y) = 4\sin x\,\sin y \;\Rightarrow\; y' = \frac{4\sin x\,\sin y}{4\cos x\,\cos y} = \tan x\,\tan y$

17. $\sqrt{xy} = 1 + x^2y \;\Rightarrow\; \frac{1}{2}(xy)^{-1/2}(xy' + y \cdot 1) = 0 + x^2y' + y \cdot 2x \;\Rightarrow\; \frac{x}{2\sqrt{xy}}\,y' + \frac{y}{2\sqrt{xy}} = x^2y' + 2xy \;\Rightarrow$

$y'\left(\frac{x}{2\sqrt{xy}} - x^2\right) = 2xy - \frac{y}{2\sqrt{xy}} \;\Rightarrow\; y'\left(\frac{x - 2x^2\sqrt{xy}}{2\sqrt{xy}}\right) = \frac{4xy\sqrt{xy} - y}{2\sqrt{xy}} \;\Rightarrow\; y' = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$

29. $2(x^2 + y^2)^2 = 25(x^2 - y^2) \;\Rightarrow\; 4(x^2 + y^2)(2x + 2y\,y') = 25(2x - 2y\,y') \;\Rightarrow$

$4(x + y\,y')(x^2 + y^2) = 25(x - y\,y') \;\Rightarrow\; 4y\,y'(x^2 + y^2) + 25yy' = 25x - 4x(x^2 + y^2) \;\Rightarrow$

$y' = \frac{25x - 4x(x^2 + y^2)}{25y + 4y(x^2 + y^2)}$. When $x = 3$ and $y = 1$, we have $y' = \frac{75 - 120}{25 + 40} = -\frac{45}{65} = -\frac{9}{13}$,

so an equation of the tangent line is $y - 1 = -\frac{9}{13}(x - 3)$ or $y = -\frac{9}{13}x + \frac{40}{13}$.

45. $y = \tan^{-1}\sqrt{x} \;\Rightarrow\; y' = \frac{1}{1 + \left(\sqrt{x}\right)^2} \cdot \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{1 + x}\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2\sqrt{x}\,(1 + x)}$

53. $y = \cos^{-1}(e^{2x}) \;\Rightarrow\; y' = -\frac{1}{\sqrt{1 - (e^{2x})^2}} \cdot \frac{d}{dx}\left(e^{2x}\right) = -\frac{2e^{2x}}{\sqrt{1 - e^{4x}}}$

69. $x^2 + 4y^2 = 5 \quad \Rightarrow \quad 2x + 4(2yy') = 0 \quad \Rightarrow \quad y' = -\dfrac{x}{4y}$. Now let h be the height of the lamp, and let (a, b) be the point of tangency of the line passing through the points $(3, h)$ and $(-5, 0)$. This line has slope $(h - 0)/[3 - (-5)] = \frac{1}{8}h$. But the slope of the tangent line through the point (a, b) can be expressed as $y' = -\dfrac{a}{4b}$, or as $\dfrac{b - 0}{a - (-5)} = \dfrac{b}{a + 5}$ [since the line passes through $(-5, 0)$ and (a, b)], so $-\dfrac{a}{4b} = \dfrac{b}{a + 5} \quad \Leftrightarrow \quad 4b^2 = -a^2 - 5a \quad \Leftrightarrow \quad a^2 + 4b^2 = -5a$. But $a^2 + 4b^2 = 5$ [since (a, b) is on the ellipse], so $5 = -5a \quad \Leftrightarrow \quad a = -1$. Then $4b^2 = -a^2 - 5a = -1 - 5(-1) = 4 \quad \Rightarrow \quad b = 1$, since the point is on the top half of the ellipse. So $\dfrac{h}{8} = \dfrac{b}{a + 5} = \dfrac{1}{-1 + 5} = \dfrac{1}{4} \quad \Rightarrow \quad h = 2$. So the lamp is located 2 units above the x-axis.