

1. The differentiation formula for logarithmic functions, $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$, is simplest when $a = e$ because $\ln e = 1$.

$$2. f(x) = \ln(x^2 + 10) \Rightarrow f'(x) = \frac{1}{x^2 + 10} \cdot \frac{d}{dx}(x^2 + 10) = \frac{2x}{x^2 + 10}$$

$$3. f(x) = \sin(\ln x) \Rightarrow f'(x) = \cos(\ln x) \cdot \frac{d}{dx} \ln x = \cos(\ln x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$$

$$4. f(x) = \ln(\sin^2 x) = \ln(\sin x)^2 = 2 \ln |\sin x| \Rightarrow f'(x) = 2 \cdot \frac{1}{\sin x} \cdot \cos x = 2 \cot x$$

$$16. y = \frac{1}{\ln x} = (\ln x)^{-1} \Rightarrow y' = -1(\ln x)^{-2} \cdot \frac{1}{x} = \frac{-1}{x(\ln x)^2}$$

$$23. y = x^2 \ln(2x) \Rightarrow y' = x^2 \cdot \frac{1}{2x} \cdot 2 + \ln(2x) \cdot (2x) = x + 2x \ln(2x) \Rightarrow$$

$$y'' = 1 + 2x \cdot \frac{1}{2x} \cdot 2 + \ln(2x) \cdot 2 = 1 + 2 + 2 \ln(2x) = 3 + 2 \ln(2x)$$

$$26. y = \ln(\sec x + \tan x) \Rightarrow y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x \Rightarrow y'' = \sec x \tan x$$

$$39. y = \frac{\sin^2 x \tan^4 x}{(x^2 + 1)^2} \Rightarrow \ln y = \ln(\sin^2 x \tan^4 x) - \ln(x^2 + 1)^2 \Rightarrow$$

$$\ln y = \ln(\sin x)^2 + \ln(\tan x)^4 - \ln(x^2 + 1)^2 \Rightarrow \ln y = 2 \ln |\sin x| + 4 \ln |\tan x| - 2 \ln(x^2 + 1) \Rightarrow$$

$$\frac{1}{y} y' = 2 \cdot \frac{1}{\sin x} \cdot \cos x + 4 \cdot \frac{1}{\tan x} \cdot \sec^2 x - 2 \cdot \frac{1}{x^2 + 1} \cdot 2x \Rightarrow y' = \frac{\sin^2 x \tan^4 x}{(x^2 + 1)^2} \left(2 \cot x + \frac{4 \sec^2 x}{\tan x} - \frac{4x}{x^2 + 1} \right)$$

$$48. y = (\ln x)^{\cos x} \Rightarrow \ln y = \cos x \ln(\ln x) \Rightarrow \frac{y'}{y} = \cos x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + (\ln \ln x)(-\sin x) \Rightarrow$$

$$y' = (\ln x)^{\cos x} \left(\frac{\cos x}{x \ln x} - \sin x \ln \ln x \right)$$