3. (a) $s = f(t) = \cos(\pi t/4) \quad \Rightarrow \quad v(t) = f'(t) = -\sin(\pi t/4) \cdot (\pi/4)$

(b) $v(3) = -\frac{\pi}{4} \sin \frac{3\pi}{4} = -\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} = -\frac{\pi\sqrt{2}}{8}$ ft/s $[\approx -0.56]$

(c) The particle is at rest when $v(t) = 0$. $\quad -\frac{\pi}{4} \sin \frac{\pi t}{4} = 0 \quad \Rightarrow \quad \sin \frac{\pi t}{4} = 0 \quad \Rightarrow \quad \frac{\pi t}{4} = \pi n \quad \Rightarrow \quad t = 0, 4, 8$ s.

(d) The particle is moving in the positive direction when $v(t) > 0$. $\quad -\frac{\pi}{4} \sin \frac{\pi t}{4} > 0 \quad \Rightarrow \quad \sin \frac{\pi t}{4} < 0 \quad \Rightarrow \quad 4 < t < 8$.

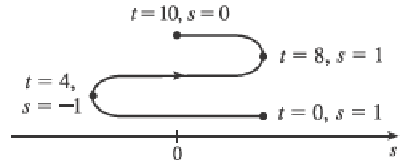
(e) From part (c), $v(t) = 0$ for $t = 0, 4, 8$. As in Exercise 1, we'll find the distance traveled in the intervals $[0, 4]$ and $[4, 8]$.

$|f(4) - f(0)| = |-1 - 1| = 2$.

$|f(8) - f(4)| = |1 - (-1)| = 2$.

The total distance is $2 + 2 = 4$ ft.

(f)

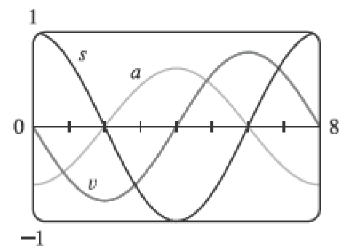


(g) $v(t) = -\dfrac{\pi}{4} \sin \dfrac{\pi t}{4} \quad \Rightarrow$

$$a(t) = v'(t) = -\frac{\pi}{4} \cos \frac{\pi t}{4} \cdot \frac{\pi}{4} = -\frac{\pi^2}{16} \cos \frac{\pi t}{4}.$$

$$a(3) = -\frac{\pi^2}{16} \cos \frac{3\pi}{4} = -\frac{\pi^2}{16}\left(-\frac{\sqrt{2}}{2}\right) = \frac{\pi^2\sqrt{2}}{32} \text{ (ft/s)/s or ft/s}^2.$$

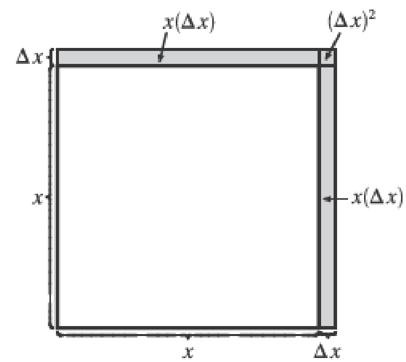
(h)



(i) The particle is speeding up when v and a have the same sign. This occurs when $0 < t < 2$ [v and a are both negative] and when $4 < t < 6$ [v and a are both positive]. It is slowing down when v and a have opposite signs; that is, when $2 < t < 4$ and when $6 < t < 8$.

11. (a) $A(x) = x^2 \quad \Rightarrow \quad A'(x) = 2x$. $A'(15) = 30\ \text{mm}^2/\text{mm}$ is the rate at which the area is increasing with respect to the side length as x reaches 15 mm.

(b) The perimeter is $P(x) = 4x$, so $A'(x) = 2x = \frac{1}{2}(4x) = \frac{1}{2}P(x)$. The figure suggests that if Δx is small, then the change in the area of the square is approximately half of its perimeter (2 of the 4 sides) times Δx. From the figure, $\Delta A = 2x\,(\Delta x) + (\Delta x)^2$. If Δx is small, then $\Delta A \approx 2x\,(\Delta x)$ and so $\Delta A/\Delta x \approx 2x$.



19. The quantity of charge is $Q(t) = t^3 - 2t^2 + 6t + 2$, so the current is $Q'(t) = 3t^2 - 4t + 6$.

(a) $Q'(0.5) = 3(0.5)^2 - 4(0.5) + 6 = 4.75$ A

(b) $Q'(1) = 3(1)^2 - 4(1) + 6 = 5$ A

The current is lowest when Q' has a minimum. $Q''(t) = 6t - 4 < 0$ when $t < \frac{2}{3}$. So the current decreases when $t < \frac{2}{3}$ and increases when $t > \frac{2}{3}$. Thus, the current is lowest at $t = \frac{2}{3}$ s.

22. (a) $[\mathrm{C}] = \dfrac{a^2kt}{akt+1} \quad\Rightarrow\quad \text{rate of reaction} = \dfrac{d[\mathrm{C}]}{dt} = \dfrac{(akt+1)(a^2k)-(a^2kt)(ak)}{(akt+1)^2} = \dfrac{a^2k(akt+1-akt)}{(akt+1)^2} = \dfrac{a^2k}{(akt+1)^2}$

(b) If $x = [\mathrm{C}]$, then $a - x = a - \dfrac{a^2kt}{akt+1} = \dfrac{a^2kt + a - a^2kt}{akt+1} = \dfrac{a}{akt+1}$.

So $k(a-x)^2 = k\left(\dfrac{a}{akt+1}\right)^2 = \dfrac{a^2k}{(akt+1)^2} = \dfrac{d[\mathrm{C}]}{dt}$ [from part (a)] $= \dfrac{dx}{dt}$.

(c) As $t \to \infty$, $[\mathrm{C}] = \dfrac{a^2kt}{akt+1} = \dfrac{(a^2kt)/t}{(akt+1)/t} = \dfrac{a^2k}{ak+(1/t)} \to \dfrac{a^2k}{ak} = a$ moles/L.

(d) As $t \to \infty$, $\dfrac{d[\mathrm{C}]}{dt} = \dfrac{a^2k}{(akt+1)^2} \to 0$.

(e) As t increases, nearly all of the reactants A and B are converted into product C. In practical terms, the reaction virtually stops.

31. (a) $A(x) = \dfrac{p(x)}{x} \quad\Rightarrow\quad A'(x) = \dfrac{xp'(x) - p(x)\cdot 1}{x^2} = \dfrac{xp'(x)-p(x)}{x^2}$.

$A'(x) > 0 \quad\Rightarrow\quad A(x)$ is increasing; that is, the average productivity increases as the size of the workforce increases.

(b) $p'(x)$ is greater than the average productivity $\quad\Rightarrow\quad p'(x) > A(x) \quad\Rightarrow\quad p'(x) > \dfrac{p(x)}{x} \quad\Rightarrow\quad xp'(x) > p(x) \quad\Rightarrow$

$xp'(x) - p(x) > 0 \quad\Rightarrow\quad \dfrac{xp'(x)-p(x)}{x^2} > 0 \quad\Rightarrow\quad A'(x) > 0$.

35. (a) If the populations are stable, then the growth rates are neither positive nor negative; that is, $\dfrac{dC}{dt} = 0$ and $\dfrac{dW}{dt} = 0$.

(b) "The caribou go extinct" means that the population is zero, or mathematically, $C = 0$.

(c) We have the equations $\dfrac{dC}{dt} = aC - bCW$ and $\dfrac{dW}{dt} = -cW + dCW$. Let $dC/dt = dW/dt = 0$, $a = 0.05$, $b = 0.001$, $c = 0.05$, and $d = 0.0001$ to obtain $0.05C - 0.001CW = 0$ (1) and $-0.05W + 0.0001CW = 0$ (2). Adding 10 times (2) to (1) eliminates the CW-terms and gives us $0.05C - 0.5W = 0 \quad\Rightarrow\quad C = 10W$. Substituting $C = 10W$ into (1) results in $0.05(10W) - 0.001(10W)W = 0 \quad\Leftrightarrow\quad 0.5W - 0.01W^2 = 0 \quad\Leftrightarrow\quad 50W - W^2 = 0 \quad\Leftrightarrow$ $W(50 - W) = 0 \quad\Leftrightarrow\quad W = 0$ or 50. Since $C = 10W$, $C = 0$ or 500. Thus, the population pairs (C, W) that lead to stable populations are $(0, 0)$ and $(500, 50)$. So it is possible for the two species to live in harmony.