10. (a) If $y(t)$ is the mass after t days and $y(0) = A$, then $y(t) = Ae^{kt}$.

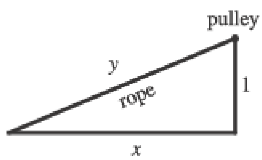
$y(1) = Ae^k = 0.945A \quad \Rightarrow \quad e^k = 0.945 \quad \Rightarrow \quad k = \ln 0.945.$

Then $Ae^{(\ln 0.945)t} = \frac{1}{2}A \quad \Leftrightarrow \quad \ln e^{(\ln 0.945)t} = \ln \frac{1}{2} \quad \Leftrightarrow \quad (\ln 0.945)t = \ln \frac{1}{2} \quad \Leftrightarrow \quad t = -\frac{\ln 2}{\ln 0.945} \approx 12.25$ years.

(b) $Ae^{(\ln 0.945)t} = 0.20A \quad \Leftrightarrow \quad (\ln 0.945)t = \ln \frac{1}{5} \quad \Leftrightarrow \quad t = -\frac{\ln 5}{\ln 0.945} \approx 28.45$ years

16. $\frac{dT}{dt} = k(T - 20)$. Let $y = T - 20$. Then $\frac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt}$. $y(0) = T(0) - 20 = 95 - 20 = 75$, so $y(t) = 75e^{kt}$. When $T(t) = 70$, $\frac{dT}{dt} = -1°\text{C/min}$. Equivalently, $\frac{dy}{dt} = -1$ when $y(t) = 50$. Thus, $-1 = \frac{dy}{dt} = ky(t) = 50k$ and $50 = y(t) = 75e^{kt}$. The first relation implies $k = -1/50$, so the second relation says $50 = 75e^{-t/50}$. Thus, $e^{-t/50} = \frac{2}{3} \quad \Rightarrow \quad -t/50 = \ln\left(\frac{2}{3}\right) \quad \Rightarrow \quad t = -50\ln\left(\frac{2}{3}\right) \approx 20.27$ min.

20.

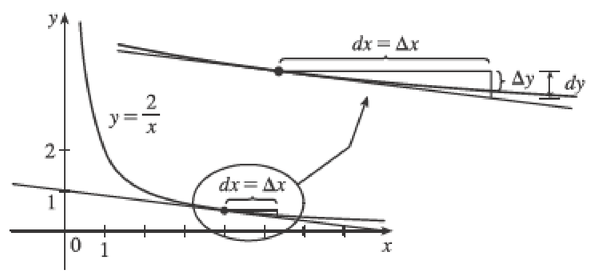


Given $\frac{dy}{dt} = -1$ m/s, find $\frac{dx}{dt}$ when $x = 8$ m. $y^2 = x^2 + 1 \quad \Rightarrow \quad 2y\,\frac{dy}{dt} = 2x\,\frac{dx}{dt} \quad \Rightarrow$ $\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt} = -\frac{y}{x}$. When $x = 8$, $y = \sqrt{65}$, so $\frac{dx}{dt} = -\frac{\sqrt{65}}{8}$. Thus, the boat approaches the dock at $\frac{\sqrt{65}}{8} \approx 1.01$ m/s.

21. $y = f(x) = 2/x,\ x = 4,\ \Delta x = 1 \quad \Rightarrow$

$\Delta y = f(5) - f(4) = \frac{2}{5} - \frac{2}{4} = -0.1$

$dy = -\frac{2}{x^2}\,dx = -\frac{2}{4^2}(1) = -0.125$



22. $y = f(x) = e^x,\ x = 0,\ \Delta x = 0.5 \quad \Rightarrow$

$\Delta y = f(0.5) - f(0) = \sqrt{e} - 1 \quad [\approx 0.65]$

$dy = e^x\,dx = e^0(0.5) = 0.5$