
Section 3.8

2. (a) By Theorem 2, $P(t) = P(0)e^{kt} = 60e^{kt}$. In 20 minutes ($\frac{1}{3}$ hour), there are 120 cells, so $P(\frac{1}{3}) = 60e^{k/3} = 120 \Rightarrow$

$$e^{k/3} = 2 \Rightarrow k/3 = \ln 2 \Rightarrow k = 3 \ln 2 = \ln(2^3) = \ln 8.$$

(b) $P(t) = 60e^{(\ln 8)t} = 60 \cdot 8^t$

(c) $P(8) = 60 \cdot 8^8 = 60 \cdot 2^{24} = 1,006,632,960$

(d) $dP/dt = kP \Rightarrow P'(8) = kP(8) = (\ln 8)P(8) \approx 2.093$ billion cells/h

(e) $P(t) = 20,000 \Rightarrow 60 \cdot 8^t = 20,000 \Rightarrow 8^t = 1000/3 \Rightarrow t \ln 8 = \ln(1000/3) \Rightarrow$

$$t = \frac{\ln(1000/3)}{\ln 8} \approx 2.79 \text{ h}$$

7. (a) If $y = [\text{N}_2\text{O}_5]$ then by Theorem 2, $\frac{dy}{dt} = -0.0005y \Rightarrow y(t) = y(0)e^{-0.0005t} = Ce^{-0.0005t}$.

(b) $y(t) = Ce^{-0.0005t} = 0.9C \Rightarrow e^{-0.0005t} = 0.9 \Rightarrow -0.0005t = \ln 0.9 \Rightarrow t = -2000 \ln 0.9 \approx 211 \text{ s}$

12. From the information given, we know that $\frac{dy}{dx} = 2y \Rightarrow y = Ce^{2x}$ by Theorem 2. To calculate C we use the point $(0, 5)$:

$$5 = Ce^{2(0)} \Rightarrow C = 5. \text{ Thus, the equation of the curve is } y = 5e^{2x}.$$

15. $\frac{dT}{dt} = k(T - 20)$. Letting $y = T - 20$, we get $\frac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt}$. $y(0) = T(0) - 20 = 5 - 20 = -15$, so

$$y(25) = y(0)e^{25k} = -15e^{25k}, \text{ and } y(25) = T(25) - 20 = 10 - 20 = -10, \text{ so } -15e^{25k} = -10 \Rightarrow e^{25k} = \frac{2}{3}. \text{ Thus,}$$

$$25k = \ln\left(\frac{2}{3}\right) \text{ and } k = \frac{1}{25} \ln\left(\frac{2}{3}\right), \text{ so } y(t) = y(0)e^{kt} = -15e^{(1/25)\ln(2/3)t}. \text{ More simply, } e^{25k} = \frac{2}{3} \Rightarrow e^k = \left(\frac{2}{3}\right)^{1/25} \Rightarrow$$

$$e^{kt} = \left(\frac{2}{3}\right)^{t/25} \Rightarrow y(t) = -15 \cdot \left(\frac{2}{3}\right)^{t/25}.$$

(a) $T(50) = 20 + y(50) = 20 - 15 \cdot \left(\frac{2}{3}\right)^{50/25} = 20 - 15 \cdot \left(\frac{2}{3}\right)^2 = 20 - \frac{20}{3} = 13.\bar{3}^\circ\text{C}$

(b) $15 = T(t) = 20 + y(t) = 20 - 15 \cdot \left(\frac{2}{3}\right)^{t/25} \Rightarrow 15 \cdot \left(\frac{2}{3}\right)^{t/25} = 5 \Rightarrow \left(\frac{2}{3}\right)^{t/25} = \frac{1}{3} \Rightarrow$

$$(t/25) \ln\left(\frac{2}{3}\right) = \ln\left(\frac{1}{3}\right) \Rightarrow t = 25 \ln\left(\frac{1}{3}\right) / \ln\left(\frac{2}{3}\right) \approx 67.74 \text{ min.}$$