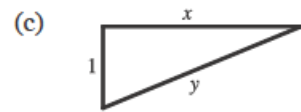


Section 3.9

11. (a) Given: a plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station.

If we let t be time (in hours) and x be the horizontal distance traveled by the plane (in mi), then we are given that $dx/dt = 500$ mi/h.

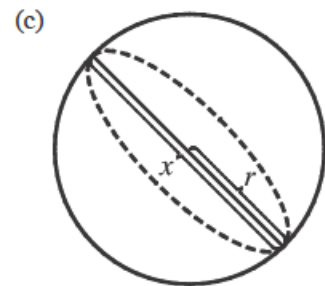
- (b) Unknown: the rate at which the distance from the plane to the station is increasing when it is 2 mi from the station. If we let y be the distance from the plane to the station, then we want to find dy/dt when $y = 2$ mi.



- (d) By the Pythagorean Theorem, $y^2 = x^2 + 1 \Rightarrow 2y (dy/dt) = 2x (dx/dt)$.

- (e) $\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{x}{y} (500)$. Since $y^2 = x^2 + 1$, when $y = 2$, $x = \sqrt{3}$, so $\frac{dy}{dt} = \frac{\sqrt{3}}{2} (500) = 250\sqrt{3} \approx 433$ mi/h.

12. (a) Given: the rate of decrease of the surface area is 1 cm²/min. If we let t be time (in minutes) and S be the surface area (in cm²), then we are given that $dS/dt = -1$ cm²/s.



- (b) Unknown: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x = 10$ cm.

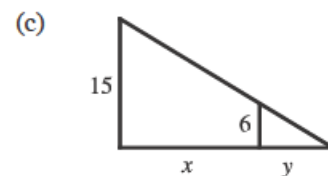
- (d) If the radius is r and the diameter $x = 2r$, then $r = \frac{1}{2}x$ and

$$S = 4\pi r^2 = 4\pi \left(\frac{1}{2}x\right)^2 = \pi x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx} \frac{dx}{dt} = 2\pi x \frac{dx}{dt}$$

- (e) $-1 = \frac{dS}{dt} = 2\pi x \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{1}{2\pi x}$. When $x = 10$, $\frac{dx}{dt} = -\frac{1}{20\pi}$. So the rate of decrease is $\frac{1}{20\pi}$ cm/min.

13. (a) Given: a man 6 ft tall walks away from a street light mounted on a 15-ft-tall pole at a rate of 5 ft/s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt = 5$ ft/s.

- (b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of his shadow (in ft), then we want to find $\frac{d}{dt}(x + y)$ when $x = 40$ ft.

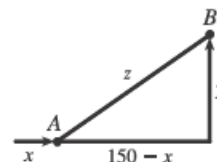


- (d) By similar triangles, $\frac{15}{6} = \frac{x + y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x$.

- (e) The tip of the shadow moves at a rate of $\frac{d}{dt}(x + y) = \frac{d}{dt}\left(x + \frac{2}{3}x\right) = \frac{5}{3} \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/s.

14. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

- (b) Unknown: the rate at which the distance between the ships is changing at 4:00 PM. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

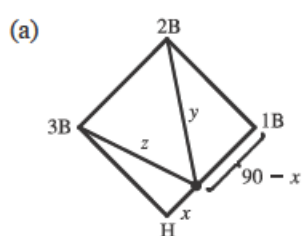


$$(d) \quad z^2 = (150 - x)^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2(150 - x) \left(-\frac{dx}{dt} \right) + 2y \frac{dy}{dt}$$

$$(e) \quad \text{At 4:00 PM, } x = 4(35) = 140 \text{ and } y = 4(25) = 100 \Rightarrow z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10,100}.$$

$$\text{So } \frac{dz}{dt} = \frac{1}{z} \left[(x - 150) \frac{dx}{dt} + y \frac{dy}{dt} \right] = \frac{-10(35) + 100(25)}{\sqrt{10,100}} = \frac{215}{\sqrt{101}} \approx 21.4 \text{ km/h.}$$

18. We are given that $\frac{dx}{dt} = 24$ ft/s.



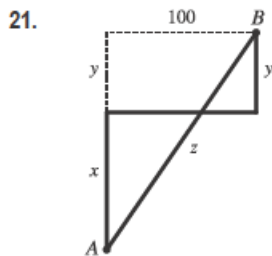
$$(a) \quad y^2 = (90 - x)^2 + 90^2 \Rightarrow 2y \frac{dy}{dt} = 2(90 - x) \left(-\frac{dx}{dt} \right). \text{ When } x = 45,$$

$$y = \sqrt{45^2 + 90^2} = 45\sqrt{5}, \text{ so } \frac{dy}{dt} = \frac{90 - x}{y} \left(-\frac{dx}{dt} \right) = \frac{45}{45\sqrt{5}} (-24) = -\frac{24}{\sqrt{5}},$$

so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

- (b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

$$z^2 = x^2 + 90^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt}. \text{ When } x = 45, z = 45\sqrt{5}, \text{ so } \frac{dz}{dt} = \frac{45}{45\sqrt{5}} (24) = \frac{24}{\sqrt{5}} \approx 10.7 \text{ ft/s.}$$



21. We are given that $\frac{dx}{dt} = 35$ km/h and $\frac{dy}{dt} = 25$ km/h. $z^2 = (x + y)^2 + 100^2 \Rightarrow$

$$2z \frac{dz}{dt} = 2(x + y) \left(\frac{dx}{dt} + \frac{dy}{dt} \right). \text{ At 4:00 PM, } x = 4(35) = 140 \text{ and } y = 4(25) = 100 \Rightarrow$$

$$z = \sqrt{(140 + 100)^2 + 100^2} = \sqrt{67,600} = 260, \text{ so}$$

$$\frac{dz}{dt} = \frac{x + y}{z} \left(\frac{dx}{dt} + \frac{dy}{dt} \right) = \frac{140 + 100}{260} (35 + 25) = \frac{720}{13} \approx 55.4 \text{ km/h.}$$

$$32. \quad PV^{1.4} = C \Rightarrow P \cdot 1.4V^{0.4} \frac{dV}{dt} + V^{1.4} \frac{dP}{dt} = 0 \Rightarrow \frac{dV}{dt} = -\frac{V^{1.4}}{P \cdot 1.4V^{0.4}} \frac{dP}{dt} = -\frac{V}{1.4P} \frac{dP}{dt}.$$

When $V = 400$, $P = 80$ and $\frac{dP}{dt} = -10$, so we have $\frac{dV}{dt} = -\frac{400}{1.4(80)}(-10) = \frac{250}{7}$. Thus, the volume is increasing at a rate of $\frac{250}{7} \approx 36$ cm³/min.

33. With $R_1 = 80$ and $R_2 = 100$, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80} + \frac{1}{100} = \frac{180}{8000} = \frac{9}{400}$, so $R = \frac{400}{9}$. Differentiating $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ with respect to t , we have $-\frac{1}{R^2} \frac{dR}{dt} = -\frac{1}{R_1^2} \frac{dR_1}{dt} - \frac{1}{R_2^2} \frac{dR_2}{dt} \Rightarrow \frac{dR}{dt} = R^2 \left(\frac{1}{R_1^2} \frac{dR_1}{dt} + \frac{1}{R_2^2} \frac{dR_2}{dt} \right)$. When $R_1 = 80$ and $R_2 = 100$, $\frac{dR}{dt} = \frac{400^2}{9^2} \left[\frac{1}{80^2} (0.3) + \frac{1}{100^2} (0.2) \right] = \frac{107}{810} \approx 0.132 \text{ } \Omega/\text{s}$.