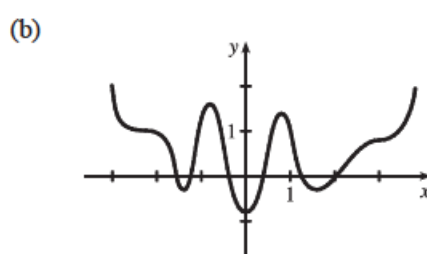
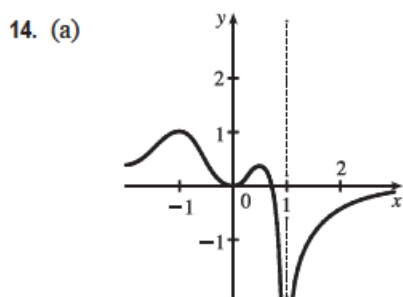
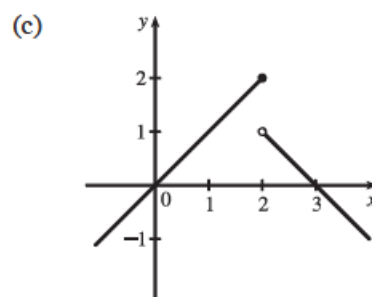
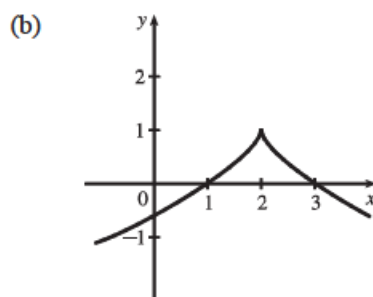
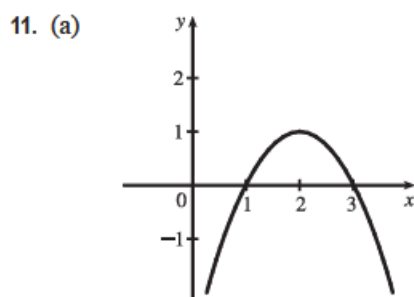


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Section 4.1

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3. Absolute maximum at  $s$ , absolute minimum at  $r$ , local maximum at  $c$ , local minima at  $b$  and  $r$ , neither a maximum nor a minimum at  $a$  and  $d$ .
4. Absolute maximum at  $r$ ; absolute minimum at  $a$ ; local maxima at  $b$  and  $r$ ; local minimum at  $d$ ; neither a maximum nor a minimum at  $c$  and  $s$ .
5. Absolute maximum value is  $f(4) = 5$ ; there is no absolute minimum value; local maximum values are  $f(4) = 5$  and  $f(6) = 4$ ; local minimum values are  $f(2) = 2$  and  $f(1) = f(5) = 3$ .
6. There is no absolute maximum value; absolute minimum value is  $g(4) = 1$ ; local maximum values are  $g(3) = 4$  and  $g(6) = 3$ ; local minimum values are  $g(2) = 2$  and  $g(4) = 1$ .



29.  $f(x) = 5x^2 + 4x \Rightarrow f'(x) = 10x + 4$ .  $f'(x) = 0 \Rightarrow x = -\frac{2}{5}$ , so  $-\frac{2}{5}$  is the only critical number.

36.  $h(p) = \frac{p-1}{p^2+4} \Rightarrow h'(p) = \frac{(p^2+4)(1) - (p-1)(2p)}{(p^2+4)^2} = \frac{p^2+4-2p^2+2p}{(p^2+4)^2} = \frac{-p^2+2p+4}{(p^2+4)^2}$ .

$h'(p) = 0 \Rightarrow p = \frac{-2 \pm \sqrt{4+16}}{-2} = 1 \pm \sqrt{5}$ . The critical numbers are  $1 \pm \sqrt{5}$ . [ $h'(p)$  exists for all real numbers.]

41.  $f(\theta) = 2 \cos \theta + \sin^2 \theta \Rightarrow f'(\theta) = -2 \sin \theta + 2 \sin \theta \cos \theta$ .  $f'(\theta) = 0 \Rightarrow 2 \sin \theta (\cos \theta - 1) = 0 \Rightarrow \sin \theta = 0$  or  $\cos \theta = 1 \Rightarrow \theta = n\pi$  [ $n$  an integer] or  $\theta = 2n\pi$ . The solutions  $\theta = n\pi$  include the solutions  $\theta = 2n\pi$ , so the critical numbers are  $\theta = n\pi$ .

44.  $f(x) = x^{-2} \ln x \Rightarrow f'(x) = x^{-2}(1/x) + (\ln x)(-2x^{-3}) = x^{-3} - 2x^{-3} \ln x = x^{-3}(1 - 2 \ln x) = \frac{1 - 2 \ln x}{x^3}$ .

$f'(x) = 0 \Rightarrow 1 - 2 \ln x = 0 \Rightarrow \ln x = \frac{1}{2} \Rightarrow x = e^{1/2} \approx 1.65$ .  $f'(0)$  does not exist, but 0 is not in the domain of  $f$ , so the only critical number is  $\sqrt{e}$ .

47.  $f(x) = 3x^2 - 12x + 5$ ,  $[0, 3]$ .  $f'(x) = 6x - 12 = 0 \Leftrightarrow x = 2$ . Applying the Closed Interval Method, we find that  $f(0) = 5$ ,  $f(2) = -7$ , and  $f(3) = -4$ . So  $f(0) = 5$  is the absolute maximum value and  $f(2) = -7$  is the absolute minimum value.

52.  $f(x) = (x^2 - 1)^3$ ,  $[-1, 2]$ .  $f'(x) = 3(x^2 - 1)^2(2x) = 6x(x + 1)^2(x - 1)^2 = 0 \Leftrightarrow x = -1, 0, 1$ .  $f(\pm 1) = 0$ ,  $f(0) = -1$ , and  $f(2) = 27$ . So  $f(2) = 27$  is the absolute maximum value and  $f(0) = -1$  is the absolute minimum value.

57.  $f(t) = 2 \cos t + \sin 2t$ ,  $[0, \pi/2]$ .

$$f'(t) = -2 \sin t + \cos 2t \cdot 2 = -2 \sin t + 2(1 - 2 \sin^2 t) = -2(2 \sin^2 t + \sin t - 1) = -2(2 \sin t - 1)(\sin t + 1).$$

$$f'(t) = 0 \Rightarrow \sin t = \frac{1}{2} \text{ or } \sin t = -1 \Rightarrow t = \frac{\pi}{6}. f(0) = 2, f(\frac{\pi}{6}) = \sqrt{3} + \frac{1}{2} \sqrt{3} = \frac{3}{2} \sqrt{3} \approx 2.60, \text{ and } f(\frac{\pi}{2}) = 0.$$

So  $f(\frac{\pi}{6}) = \frac{3}{2} \sqrt{3}$  is the absolute maximum value and  $f(\frac{\pi}{2}) = 0$  is the absolute minimum value.

62.  $f(x) = e^{-x} - e^{-2x}$ ,  $[0, 1]$ .  $f'(x) = e^{-x}(-1) - e^{-2x}(-2) = \frac{2}{e^{2x}} - \frac{1}{e^x} = \frac{2 - e^x}{e^{2x}} = 0 \Leftrightarrow e^x = 2 \Leftrightarrow$

$$x = \ln 2 \approx 0.69. f(0) = 0, f(\ln 2) = e^{-\ln 2} - e^{-2 \ln 2} = (e^{\ln 2})^{-1} - (e^{\ln 2})^{-2} = 2^{-1} - 2^{-2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4},$$

$f(1) = e^{-1} - e^{-2} \approx 0.233$ . So  $f(\ln 2) = \frac{1}{4}$  is the absolute maximum value and  $f(0) = 0$  is the absolute minimum value.