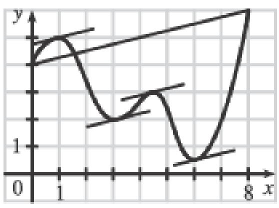
3. $f(x)=\sqrt{x}-\frac{1}{3}x$, $[0,9]$. f, being the difference of a root function and a polynomial, is continuous and differentiable on $[0,\infty)$, so it is continuous on $[0,9]$ and differentiable on $(0,9)$. Also, $f(0)=0=f(9)$. $f'(c)=0 \iff$ $\dfrac{1}{2\sqrt{c}}-\dfrac{1}{3}=0 \iff 2\sqrt{c}=3 \iff \sqrt{c}=\dfrac{3}{2} \Rightarrow c=\dfrac{9}{4}$, which is in the open interval $(0,9)$, so $c=\dfrac{9}{4}$ satisfies the conclusion of Rolle's Theorem.

4. $f(x)=\cos 2x$, $[\pi/8,7\pi/8]$. f, being the composite of the cosine function and the polynomial $2x$, is continuous and differentiable on $\mathbb{R}$, so it is continuous on $[\pi/8,7\pi/8]$ and differentiable on $(\pi/8,7\pi/8)$. Also, $f\left(\frac{\pi}{8}\right)=\frac{1}{2}\sqrt{2}=f\left(\frac{7\pi}{8}\right)$. $f'(c)=0 \iff -2\sin 2c=0 \iff \sin 2c=0 \iff 2c=\pi n \iff c=\frac{\pi}{2}n$. If $n=1$, then $c=\frac{\pi}{2}$, which is in the open interval $(\pi/8,7\pi/8)$, so $c=\frac{\pi}{2}$ satisfies the conclusion of Rolle's Theorem.

5. $f(x)=1-x^{2/3}$. $f(-1)=1-(-1)^{2/3}=1-1=0=f(1)$. $f'(x)=-\frac{2}{3}x^{-1/3}$, so $f'(c)=0$ has no solution. This does not contradict Rolle's Theorem, since $f'(0)$ does not exist, and so f is not differentiable on $(-1,1)$.

7. $\dfrac{f(8)-f(0)}{8-0}=\dfrac{6-4}{8}=\dfrac{1}{4}$. The values of c which satisfy $f'(c)=\frac{1}{4}$ seem to be about $c=0.8, 3.2, 4.4$, and 6.1.



13. $f(x)=e^{-2x}$, $[0,3]$. f is continuous and differentiable on $\mathbb{R}$, so it is continuous on $[0,3]$ and differentiable on $(0,3)$.

$$f'(c)=\frac{f(b)-f(a)}{b-a} \iff -2e^{-2c}=\frac{e^{-6}-e^{0}}{3-0} \iff e^{-2c}=\frac{1-e^{-6}}{6} \iff -2c=\ln\left(\frac{1-e^{-6}}{6}\right) \iff$$

$c=-\dfrac{1}{2}\ln\left(\dfrac{1-e^{-6}}{6}\right)\approx 0.897$, which is in $(0,3)$.

14. $f(x)=\dfrac{x}{x+2}$, $[1,4]$. f is continuous on $[1,4]$ and differentiable on $(1,4)$. $f'(c)=\dfrac{f(b)-f(a)}{b-a} \iff$ $\dfrac{2}{(c+2)^2}=\dfrac{\frac{2}{3}-\frac{1}{3}}{4-1} \iff (c+2)^2=18 \iff c=-2\pm 3\sqrt{2}$. $-2+3\sqrt{2}\approx 2.24$ is in $(1,4)$.

15. $f(x)=(x-3)^{-2} \Rightarrow f'(x)=-2\,(x-3)^{-3}$. $f(4)-f(1)=f'(c)(4-1) \Rightarrow \dfrac{1}{1^2}-\dfrac{1}{(-2)^2}=\dfrac{-2}{(c-3)^3}\cdot 3 \Rightarrow$ $\dfrac{3}{4}=\dfrac{-6}{(c-3)^3} \Rightarrow (c-3)^3=-8 \Rightarrow c-3=-2 \Rightarrow c=1$, which is not in the open interval $(1,4)$. This does not contradict the Mean Value Theorem since f is not continuous at $x=3$.

18. Let $f(x) = 2x - 1 - \sin x$. Then $f(0) = -1 < 0$ and $f(\pi/2) = \pi - 2 > 0$. f is the sum of the polynomial $2x - 1$ and the scalar multiple $(-1) \cdot \sin x$ of the trigonometric function $\sin x$, so f is continuous (and differentiable) for all x. By the Intermediate Value Theorem, there is a number c in $(0, \pi/2)$ such that $f(c) = 0$. Thus, the given equation has at least one real root. If the equation has distinct real roots a and b with $a < b$, then $f(a) = f(b) = 0$. Since f is continuous on $[a, b]$ and differentiable on (a, b), Rolle's Theorem implies that there is a number r in (a, b) such that $f'(r) = 0$. But $f'(r) = 2 - \cos r > 0$ since $\cos r \leq 1$. This contradiction shows that the given equation can't have two distinct real roots, so it has exactly one real root.

31. For $x > 0$, $f(x) = g(x)$, so $f'(x) = g'(x)$. For $x < 0$, $f'(x) = (1/x)' = -1/x^2$ and $g'(x) = (1 + 1/x)' = -1/x^2$, so again $f'(x) = g'(x)$. However, the domain of $g(x)$ is not an interval [it is $(-\infty, 0) \cup (0, \infty)$] so we cannot conclude that $f - g$ is constant (in fact it is not).