2. (a) f is increasing on $(0, 1)$ and $(3, 7)$.

(b) f is decreasing on $(1, 3)$.

(c) f is concave upward on $(2, 4)$ and $(5, 7)$.

(d) f is concave downward on $(0, 2)$ and $(4, 5)$.

(e) The points of inflection are $(2, 2)$, $(4, 3)$, and $(5, 4)$.

6. (a) $f'(x) > 0$ and f is increasing on $(0, 1)$ and $(3, 5)$. $f'(x) < 0$ and f is decreasing on $(1, 3)$ and $(5, 6)$.

(b) Since $f'(x) = 0$ at $x = 1$ and $x = 5$ and f' changes from positive to negative at both values, f changes from increasing to decreasing and has local maxima at $x = 1$ and $x = 5$. Since $f'(x) = 0$ at $x = 3$ and f' changes from negative to positive there, f changes from decreasing to increasing and has a local minimum at $x = 3$.

9. (a) $f(x) = 2x^3 + 3x^2 - 36x \quad \Rightarrow \quad f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x + 3)(x - 2)$.

We don't need to include the "6" in the chart to determine the sign of $f'(x)$.

Interval	$x + 3$	$x - 2$	$f'(x)$	f
$x < -3$	$-$	$-$	$+$	increasing on $(-\infty, -3)$
$-3 < x < 2$	$+$	$-$	$-$	decreasing on $(-3, 2)$
$x > 2$	$+$	$+$	$+$	increasing on $(2, \infty)$

(b) f changes from increasing to decreasing at $x = -3$ and from decreasing to increasing at $x = 2$. Thus, $f(-3) = 81$ is a local maximum value and $f(2) = -44$ is a local minimum value.

(c) $f'(x) = 6x^2 + 6x - 36 \quad \Rightarrow \quad f''(x) = 12x + 6$. $f''(x) = 0$ at $x = -\frac{1}{2}$, $f''(x) > 0 \quad \Leftrightarrow \quad x > -\frac{1}{2}$, and $f''(x) < 0 \quad \Leftrightarrow \quad x < -\frac{1}{2}$. Thus, f is concave upward on $\left(-\frac{1}{2}, \infty\right)$ and concave downward on $\left(-\infty, -\frac{1}{2}\right)$. There is an inflection point at $\left(-\frac{1}{2}, f\left(-\frac{1}{2}\right)\right) = \left(-\frac{1}{2}, \frac{37}{2}\right)$.

13. (a) $f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$. $f'(x) = \cos x - \sin x = 0 \Rightarrow \cos x = \sin x \Rightarrow 1 = \dfrac{\sin x}{\cos x} \Rightarrow$ $\tan x = 1 \Rightarrow x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$. Thus, $f'(x) > 0 \Leftrightarrow \cos x - \sin x > 0 \Leftrightarrow \cos x > \sin x \Leftrightarrow 0 < x < \frac{\pi}{4}$ or $\frac{5\pi}{4} < x < 2\pi$ and $f'(x) < 0 \Leftrightarrow \cos x < \sin x \Leftrightarrow \frac{\pi}{4} < x < \frac{5\pi}{4}$. So f is increasing on $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{5\pi}{4}, 2\pi\right)$ and f is decreasing on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

(b) f changes from increasing to decreasing at $x = \frac{\pi}{4}$ and from decreasing to increasing at $x = \frac{5\pi}{4}$. Thus, $f\left(\frac{\pi}{4}\right) = \sqrt{2}$ is a local maximum value and $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$ is a local minimum value.

(c) $f''(x) = -\sin x - \cos x = 0 \Rightarrow -\sin x = \cos x \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$. Divide the interval $(0, 2\pi)$ into subintervals with these numbers as endpoints and complete a second derivative chart.

Interval	$f''(x) = -\sin x - \cos x$	Concavity
$\left(0, \frac{3\pi}{4}\right)$	$f''\left(\frac{\pi}{2}\right) = -1 < 0$	downward
$\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$	$f''(\pi) = 1 > 0$	upward
$\left(\frac{7\pi}{4}, 2\pi\right)$	$f''\left(\frac{11\pi}{6}\right) = \frac{1}{2} - \frac{1}{2}\sqrt{3} < 0$	downward

There are inflection points at $\left(\frac{3\pi}{4}, 0\right)$ and $\left(\frac{7\pi}{4}, 0\right)$.

20. $f(x) = \dfrac{x}{x^2+4} \Rightarrow f'(x) = \dfrac{(x^2+4)\cdot 1 - x(2x)}{(x^2+4)^2} = \dfrac{4-x^2}{(x^2+4)^2} = \dfrac{(2+x)(2-x)}{(x^2+4)^2}$.

First Derivative Test: $f'(x) > 0 \Rightarrow -2 < x < 2$ and $f'(x) < 0 \Rightarrow x > 2$ or $x < -2$. Since f' changes from positive to negative at $x = 2$, $f(2) = \frac{1}{4}$ is a local maximum value; and since f' changes from negative to positive at $x = -2$, $f(-2) = -\frac{1}{4}$ is a local minimum value.

Second Derivative Test:

$$f''(x) = \frac{(x^2+4)^2(-2x) - (4-x^2)\cdot 2(x^2+4)(2x)}{[(x^2+4)^2]^2} = \frac{-2x(x^2+4)\left[(x^2+4) + 2(4-x^2)\right]}{(x^2+4)^4} = \frac{-2x(12-x^2)}{(x^2+4)^3}.$$

$f'(x) = 0 \Leftrightarrow x = \pm 2$. $f''(-2) = \frac{1}{16} > 0 \Rightarrow f(-2) = -\frac{1}{4}$ is a local minimum value.

$f''(2) = -\frac{1}{16} < 0 \Rightarrow f(2) = \frac{1}{4}$ is a local maximum value.

Preference: Since calculating the second derivative is fairly difficult, the First Derivative Test is easier to use for this function.

25. $f'(0) = f'(2) = f'(4) = 0 \Rightarrow$ horizontal tangents at $x = 0, 2, 4$.

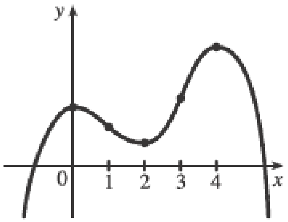
$f'(x) > 0$ if $x < 0$ or $2 < x < 4 \Rightarrow f$ is increasing on $(-\infty, 0)$ and $(2, 4)$.

$f'(x) < 0$ if $0 < x < 2$ or $x > 4 \Rightarrow f$ is decreasing on $(0, 2)$ and $(4, \infty)$.

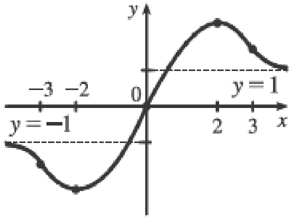
$f''(x) > 0$ if $1 < x < 3 \Rightarrow f$ is concave upward on $(1, 3)$.

$f''(x) < 0$ if $x < 1$ or $x > 3 \Rightarrow f$ is concave downward on $(-\infty, 1)$ and $(3, \infty)$.

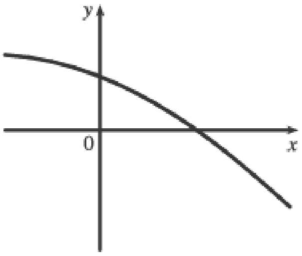
There are inflection points when $x = 1$ and 3.



28. $f'(x) > 0$ if $|x| < 2 \quad \Rightarrow \quad f$ is increasing on $(-2, 2)$. $f'(x) < 0$ if $|x| > 2 \quad \Rightarrow$ f is decreasing on $(-\infty, -2)$ and $(2, \infty)$. $f'(2) = 0$, so f has a horizontal tangent (and local maximum) at $x = 2$. $\lim\limits_{x\to\infty} f(x) = 1 \quad \Rightarrow \quad y = 1$ is a horizontal asymptote. $f(-x) = -f(x) \quad \Rightarrow \quad f$ is an odd function (its graph is symmetric about the origin). Finally, $f''(x) < 0$ if $0 < x < 3$ and $f''(x) > 0$ if $x > 3$, so f is CD on $(0, 3)$ and CU on $(3, \infty)$.



29. The function must be always decreasing (since the first derivative is always negative) and concave downward (since the second derivative is always negative).

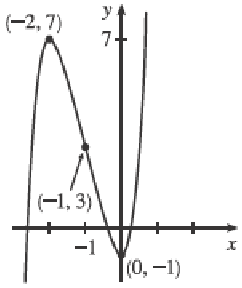


37. (a) $h(x) = (x+1)^5 - 5x - 2 \quad \Rightarrow \quad h'(x) = 5(x+1)^4 - 5$. $h'(x) = 0 \quad \Leftrightarrow \quad 5(x+1)^4 = 5 \quad \Leftrightarrow \quad (x+1)^4 = 1 \quad \Rightarrow$ $(x+1)^2 = 1 \quad \Rightarrow \quad x + 1 = 1$ or $x + 1 = -1 \quad \Rightarrow \quad x = 0$ or $x = -2$. $h'(x) > 0 \quad \Leftrightarrow \quad x < -2$ or $x > 0$ and $h'(x) < 0 \quad \Leftrightarrow \quad -2 < x < 0$. So h is increasing on $(-\infty, -2)$ and $(0, \infty)$ and h is decreasing on $(-2, 0)$.

(b) $h(-2) = 7$ is a local maximum value and $h(0) = -1$ is a local minimum value.

(c) $h''(x) = 20(x+1)^3 = 0 \quad \Leftrightarrow \quad x = -1$. $h''(x) > 0 \quad \Leftrightarrow \quad x > -1$ and $h''(x) < 0 \quad \Leftrightarrow \quad x < -1$, so h is CU on $(-1, \infty)$ and h is CD on $(-\infty, -1)$. There is a point of inflection at $(-1, h(-1)) = (-1, 3)$.

(d)

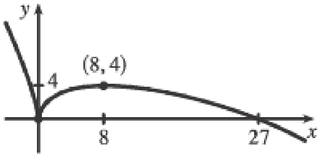


40. (a) $B(x) = 3x^{2/3} - x \quad \Rightarrow \quad B'(x) = 2x^{-1/3} - 1 = \dfrac{2}{\sqrt[3]{x}} - 1 = \dfrac{2 - \sqrt[3]{x}}{\sqrt[3]{x}}$. $B'(x) > 0$ if $0 < x < 8$ and $B'(x) < 0$ if $x < 0$ or $x > 8$, so B is decreasing on $(-\infty, 0)$ and $(8, \infty)$, and B is increasing on $(0, 8)$.

(b) $B(0) = 0$ is a local minimum value. $B(8) = 4$ is a local maximum value.

(c) $B''(x) = -\frac{2}{3}x^{-4/3} = \dfrac{-2}{3x^{4/3}}$, so $B''(x) < 0$ for all $x \neq 0$. B is concave downward on $(-\infty, 0)$ and $(0, \infty)$. There is no inflection point.

(d)



53. The nonnegative factors $(x+1)^2$ and $(x-6)^4$ do not affect the sign of $f'(x) = (x+1)^2(x-3)^5(x-6)^4$. So $f'(x) > 0 \quad \Rightarrow \quad (x-3)^5 > 0 \quad \Rightarrow \quad x - 3 > 0 \quad \Rightarrow \quad x > 3$. Thus, f is increasing on the interval $(3, \infty)$.

80. $f(x) = x^4 \quad \Rightarrow \quad f'(x) = 4x^3 \quad \Rightarrow \quad f''(x) = 12x^2 \quad \Rightarrow \quad f''(0) = 0$. For $x < 0$, $f''(x) > 0$, so f is CU on $(-\infty, 0)$; for $x > 0$, $f''(x) > 0$, so f is also CU on $(0, \infty)$. Since f does not change concavity at 0, $(0, 0)$ is not an inflection point.