3. (a) When x is near a, $f(x)$ is near 0 and $p(x)$ is large, so $f(x) - p(x)$ is large negative. Thus, $\lim_{x\to a} [f(x) - p(x)] = -\infty$.

(b) $\lim_{x\to a} [p(x) - q(x)]$ is an indeterminate form of type $\infty - \infty$.

(c) When x is near a, $p(x)$ and $q(x)$ are both large, so $p(x) + q(x)$ is large. Thus, $\lim_{x\to a} [p(x) + q(x)] = \infty$.

4. (a) $\lim_{x\to a} [f(x)]^{g(x)}$ is an indeterminate form of type 0^0.

(b) If $y = [f(x)]^{p(x)}$, then $\ln y = p(x) \ln f(x)$. When x is near a, $p(x) \to \infty$ and $\ln f(x) \to -\infty$, so $\ln y \to -\infty$.

Therefore, $\lim_{x\to a} [f(x)]^{p(x)} = \lim_{x\to a} y = \lim_{x\to a} e^{\ln y} = 0$, provided f^p is defined.

(c) $\lim_{x\to a} [h(x)]^{p(x)}$ is an indeterminate form of type 1^∞.

(d) $\lim_{x\to a} [p(x)]^{f(x)}$ is an indeterminate form of type ∞^0.

(e) If $y = [p(x)]^{q(x)}$, then $\ln y = q(x) \ln p(x)$. When x is near a, $q(x) \to \infty$ and $\ln p(x) \to \infty$, so $\ln y \to \infty$. Therefore,

$\lim_{x\to a} [p(x)]^{q(x)} = \lim_{x\to a} y = \lim_{x\to a} e^{\ln y} = \infty$.

(f) $\lim_{x\to a} \sqrt[q(x)]{p(x)} = \lim_{x\to a} [p(x)]^{1/q(x)}$ is an indeterminate form of type ∞^0.

5. This limit has the form $\frac{0}{0}$. We can simply factor and simplify to evaluate the limit.

$$\lim_{x\to 1} \frac{x^2 - 1}{x^2 - x} = \lim_{x\to 1} \frac{(x+1)(x-1)}{x(x-1)} = \lim_{x\to 1} \frac{x+1}{x} = \frac{1+1}{1} = 2$$

6. This limit has the form $\frac{0}{0}$. $\lim_{x\to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2} (x+3) = 2 + 3 = 5$

17. $\lim_{x\to 0^+} [(\ln x)/x] = -\infty$ since $\ln x \to -\infty$ as $x \to 0^+$ and dividing by small values of x just increases the magnitude of the quotient $(\ln x)/x$. L'Hospital's Rule does not apply.

18. This limit has the form $\frac{\infty}{\infty}$. $\lim_{x\to\infty} \frac{\ln \ln x}{x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x \ln x} = 0$

27. This limit has the form $\frac{0}{0}$. $\lim_{x\to 0} \frac{\sin^{-1} x}{x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{1/\sqrt{1-x^2}}{1} = \lim_{x\to 0} \frac{1}{\sqrt{1-x^2}} = \frac{1}{1} = 1$

36. This limit has the form $\frac{0}{0}$. $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{e^x + e^{-x}}{\cos x} = \frac{1+1}{1} = 2$

37. This limit has the form $\frac{0}{0}$. $\lim_{x\to 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4} \overset{\text{H}}{=} \lim_{x\to 0} \frac{-\sin x + x}{4x^3} \overset{\text{H}}{=} \lim_{x\to 0} \frac{-\cos x + 1}{12x^2} \overset{\text{H}}{=} \lim_{x\to 0} \frac{\sin x}{24x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{\cos x}{24} = \frac{1}{24}$

43. This limit has the form $\infty \cdot 0$. $\lim_{x\to\infty} x^3 e^{-x^2} = \lim_{x\to\infty} \frac{x^3}{e^{x^2}} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{3x^2}{2xe^{x^2}} = \lim_{x\to\infty} \frac{3x}{2e^{x^2}} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{3}{4xe^{x^2}} = 0$

46. This limit has the form $\infty \cdot 0$.

$$\lim_{x\to\infty} x\tan(1/x) = \lim_{x\to\infty} \frac{\tan(1/x)}{1/x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{\sec^2(1/x)(-1/x^2)}{-1/x^2} = \lim_{x\to\infty} \sec^2(1/x) = 1^2 = 1$$

53. $y = x^{x^2} \Rightarrow \ln y = x^2 \ln x$, so $\lim_{x\to 0+} \ln y = \lim_{x\to 0+} x^2 \ln x = \lim_{x\to 0+} \frac{\ln x}{1/x^2} \overset{\text{H}}{=} \lim_{x\to 0+} \frac{1/x}{-2/x^3} = \lim_{x\to 0+} \left(-\frac{1}{2}x^2\right) = 0 \Rightarrow$

$\lim_{x\to 0+} x^{x^2} = \lim_{x\to 0+} e^{\ln y} = e^0 = 1$.

71. $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{2}(x^2+1)^{-1/2}(2x)} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x}$. Repeated applications of l'Hospital's Rule result in the original limit or the limit of the reciprocal of the function. Another method is to try dividing the numerator and denominator by x: $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{x/x}{\sqrt{x^2/x^2+1/x^2}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+1/x^2}} = \frac{1}{1} = 1$