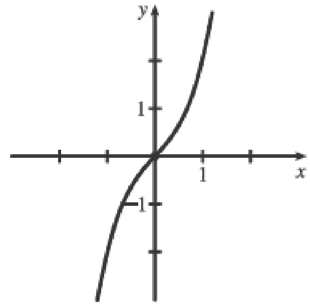
1. $y = f(x) = x^3 + x = x(x^2+1)$ **A.** f is a polynomial, so $D = \mathbb{R}$. **B.** x-intercept $= 0$, y-intercept $= f(0) = 0$ **C.** $f(-x) = -f(x)$, so f is odd; the curve is symmetric about the origin. **D.** f is a polynomial, so there is no asymptote. **E.** $f'(x) = 3x^2 + 1 > 0$, so f is increasing on $(-\infty, \infty)$. **F.** There is no critical number and hence, no local maximum or minimum value. **G.** $f''(x) = 6x > 0$ on $(0, \infty)$ and $f''(x) < 0$ on $(-\infty, 0)$, so f is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. Since the concavity changes at $x = 0$, there is an inflection point at $(0, 0)$.

H.

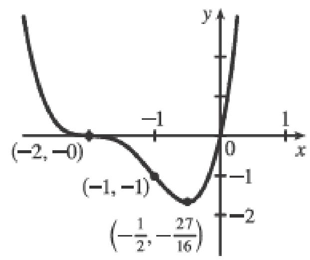


6. $y = f(x) = x(x+2)^3$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \Leftrightarrow x = -2, 0$ **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 3x(x+2)^2 + (x+2)^3 = (x+2)^2\,[3x + (x+2)] = (x+2)^2(4x+2)$. $f'(x) > 0 \Leftrightarrow x > -\frac{1}{2}$, and $f'(x) < 0 \Leftrightarrow x < -2$ or $-2 < x < -\frac{1}{2}$, so f is increasing on $\left(-\frac{1}{2}, \infty\right)$ and decreasing on $(-\infty, -2)$ and $\left(-2, -\frac{1}{2}\right)$. [Hence f is decreasing on $\left(-\infty, -\frac{1}{2}\right)$ by the analogue of Exercise 4.3.65 for decreasing functions.] **F.** Local minimum value $f\left(-\frac{1}{2}\right) = -\frac{27}{16}$, no local maximum

G.
$$\begin{aligned} f''(x) &= (x+2)^2(4) + (4x+2)(2)(x+2) \\ &= 2(x+2)[(x+2)(2) + 4x + 2] \\ &= 2(x+2)(6x+6) = 12(x+1)(x+2) \end{aligned}$$

$f''(x) < 0 \Leftrightarrow -2 < x < -1$, so f is CD on $(-2, -1)$ and CU on $(-\infty, -2)$ and $(-1, \infty)$. IP at $(-2, 0)$ and $(-1, -1)$

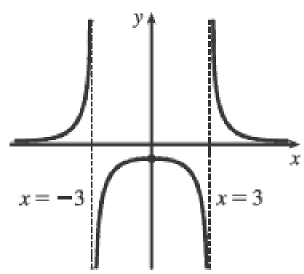
H.



11. $y = f(x) = 1/(x^2 - 9)$ **A.** $D = \{x \mid x \neq \pm 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ **B.** y-intercept $= f(0) = -\frac{1}{9}$, no x-intercept **C.** $f(-x) = f(x) \Rightarrow f$ is even; the curve is symmetric about the y-axis. **D.** $\lim\limits_{x \to \pm\infty} \dfrac{1}{x^2 - 9} = 0$, so $y = 0$ is a HA. $\lim\limits_{x \to 3^-} \dfrac{1}{x^2-9} = -\infty$, $\lim\limits_{x \to 3^+} \dfrac{1}{x^2-9} = \infty$, $\lim\limits_{x \to -3^-} \dfrac{1}{x^2-9} = \infty$, $\lim\limits_{x \to -3^+} \dfrac{1}{x^2-9} = -\infty$, so $x = 3$ and $x = -3$ are VA. **E.** $f'(x) = -\dfrac{2x}{(x^2-9)^2} > 0 \Leftrightarrow x < 0$ $(x \neq -3)$ so f is increasing on $(-\infty, -3)$ and $(-3, 0)$ and decreasing on $(0, 3)$ and $(3, \infty)$. **F.** Local maximum value $f(0) = -\frac{1}{9}$.

G. $y'' = \dfrac{-2(x^2-9)^2 + (2x)2(x^2-9)(2x)}{(x^2-9)^4} = \dfrac{6(x^2+3)}{(x^2-9)^3} > 0 \Leftrightarrow$ $x^2 > 9 \Leftrightarrow x > 3$ or $x < -3$, so f is CU on $(-\infty, -3)$ and $(3, \infty)$ and CD on $(-3, 3)$. No IP

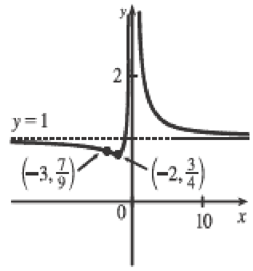
H.



16. $y = f(x) = 1 + \frac{1}{x} + \frac{1}{x^2} = \frac{x^2 + x + 1}{x^2}$ **A.** $D = (-\infty, 0) \cup (0, \infty)$ **B.** y-intercept: none $[x \neq 0]$; x-intercepts: $f(x) = 0 \Leftrightarrow x^2 + x + 1 = 0$, there is no real solution, and hence, no x-intercept **C.** No symmetry

D. $\lim_{x\to\pm\infty}\left(1 + \frac{1}{x} + \frac{1}{x^2}\right) = 1$, so $y = 1$ is a HA. $\lim_{x\to 0} f(x) = \infty$, so $x = 0$ is a VA. **E.** $f'(x) = -\frac{1}{x^2} - \frac{2}{x^3} = \frac{-x-2}{x^3}$. $f'(x) > 0 \Leftrightarrow -2 < x < 0$ and $f'(x) < 0 \Leftrightarrow x < -2$ or $x > 0$, so f is increasing on $(-2, 0)$ and decreasing on $(-\infty, -2)$ and $(0, \infty)$. **F.** Local minimum value $f(-2) = \frac{3}{4}$; no local maximum **G.** $f''(x) = \frac{2}{x^3} + \frac{6}{x^4} = \frac{2x+6}{x^4}$. $f''(x) < 0 \Leftrightarrow x < -3$ and $f''(x) > 0 \Leftrightarrow -3 < x < 0$ and $x > 0$, so f is CD on $(-\infty, -3)$ and CU on $(-3, 0)$ and $(0, \infty)$. IP at $\left(-3, \frac{7}{9}\right)$

H.



21. $y = f(x) = \sqrt{x^2 + x - 2} = \sqrt{(x+2)(x-1)}$ **A.** $D = \{x \mid (x+2)(x-1) \geq 0\} = (-\infty, -2] \cup [1, \infty)$

B. y-intercept: none; x-intercepts: -2 and 1 **C.** No symmetry **D.** No asymptote

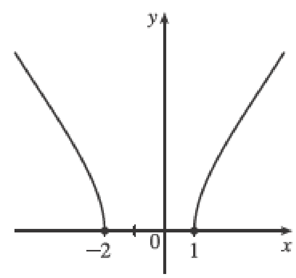
E. $f'(x) = \frac{1}{2}(x^2 + x - 2)^{-1/2}(2x + 1) = \frac{2x+1}{2\sqrt{x^2+x-2}}$, $f'(x) = 0$ if $x = -\frac{1}{2}$, but $-\frac{1}{2}$ is not in the domain.

$f'(x) > 0 \Rightarrow x > -\frac{1}{2}$ and $f'(x) < 0 \Rightarrow x < -\frac{1}{2}$, so (considering the domain) f is increasing on $(1, \infty)$ and f is decreasing on $(-\infty, -2)$. **F.** No local extrema

G.
$$f''(x) = \frac{2(x^2+x-2)^{1/2}(2) - (2x+1)\cdot 2\cdot \frac{1}{2}(x^2+x-2)^{-1/2}(2x+1)}{\left(2\sqrt{x^2+x-2}\right)^2}$$
$$= \frac{(x^2+x-2)^{-1/2}\left[4(x^2+x-2) - (4x^2+4x+1)\right]}{4(x^2+x-2)}$$
$$= \frac{-9}{4(x^2+x-2)^{3/2}} < 0$$

so f is CD on $(-\infty, -2)$ and $(1, \infty)$. No IP

H.



26. $y = f(x) = x/\sqrt{x^2-1}$ **A.** $D = (-\infty, -1) \cup (1, \infty)$ **B.** No intercepts **C.** $f(-x) = -f(x)$, so f is odd; the graph is symmetric about the origin. **D.** $\lim_{x\to\infty} \frac{x}{\sqrt{x^2-1}} = 1$ and $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-1}} = -1$, so $y = \pm 1$ are HA.

$\lim_{x\to 1^+} f(x) = +\infty$ and $\lim_{x\to -1^-} f(x) = -\infty$, so $x = \pm 1$ are VA.

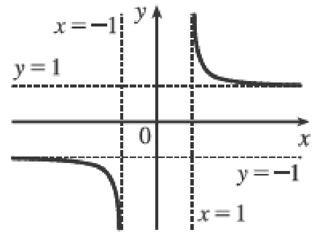
E. $f'(x) = \dfrac{\sqrt{x^2-1} - x \cdot \dfrac{x}{\sqrt{x^2-1}}}{[(x^2-1)^{1/2}]^2} = \dfrac{x^2-1-x^2}{(x^2-1)^{3/2}} = \dfrac{-1}{(x^2-1)^{3/2}} < 0$, so f is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

F. No extreme values

G. $f''(x) = (-1)\left(-\frac{3}{2}\right)(x^2-1)^{-5/2} \cdot 2x = \dfrac{3x}{(x^2-1)^{5/2}}$.

$f''(x) < 0$ on $(-\infty, -1)$ and $f''(x) > 0$ on $(1, \infty)$, so f is CD on $(-\infty, -1)$ and CU on $(1, \infty)$. No IP

H.



31. $y = f(x) = 3\sin x - \sin^3 x$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \Rightarrow \sin x\,(3 - \sin^2 x) = 0 \Rightarrow \sin x = 0$ [since $\sin^2 x \le 1 < 3$] $\Rightarrow x = n\pi$, n an integer.

C. $f(-x) = -f(x)$, so f is odd; the graph (shown for $-2\pi \le x \le 2\pi$) is symmetric about the origin and periodic with period 2π. **D.** No asymptote **E.** $f'(x) = 3\cos x - 3\sin^2 x\,\cos x = 3\cos x\,(1 - \sin^2 x) = 3\cos^3 x$.

$f'(x) > 0 \Leftrightarrow \cos x > 0 \Leftrightarrow x \in \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$ for each integer n, and $f'(x) < 0 \Leftrightarrow \cos x < 0 \Leftrightarrow x \in \left(2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right)$ for each integer n. Thus, f is increasing on $\left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$ for each integer n, and f is decreasing on $\left(2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right)$ for each integer n.

F. f has local maximum values $f\left(2n\pi + \frac{\pi}{2}\right) = 2$ and local minimum values $f\left(2n\pi + \frac{3\pi}{2}\right) = -2$.

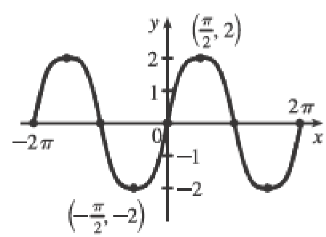
G. $f''(x) = -9\sin x\,\cos^2 x = -9\sin x\,(1 - \sin^2 x) = -9\sin x\,(1 - \sin x)(1 + \sin x)$.

$f''(x) < 0 \Leftrightarrow \sin x > 0$ and $\sin x \ne \pm 1 \Leftrightarrow x \in \left(2n\pi, 2n\pi + \frac{\pi}{2}\right) \cup \left(2n\pi + \frac{\pi}{2}, 2n\pi + \pi\right)$ for some integer n.

$f''(x) > 0 \Leftrightarrow \sin x < 0$ and $\sin x \ne \pm 1 \Leftrightarrow x \in \left((2n-1)\pi, (2n-1)\pi + \frac{\pi}{2}\right) \cup \left((2n-1)\pi + \frac{\pi}{2}, 2n\pi\right)$

for some integer n. Thus, f is CD on the intervals $\left(2n\pi, \left(2n + \frac{1}{2}\right)\pi\right)$ and $\left(\left(2n + \frac{1}{2}\right)\pi, (2n+1)\pi\right)$ [hence CD on the intervals $(2n\pi, (2n+1)\pi)$] for each integer n, and f is CU on the intervals $\left((2n-1)\pi, \left(2n - \frac{1}{2}\right)\pi\right)$ and $\left(\left(2n - \frac{1}{2}\right)\pi, 2n\pi\right)$ [hence CU on the intervals $((2n-1)\pi, 2n\pi)$] for each integer n. f has inflection points at $(n\pi, 0)$ for each integer n.

H.



36. $y = f(x) = \sec x + \tan x,\ 0 < x < \pi/2$ **A.** $D = \left(0, \frac{\pi}{2}\right)$ **B.** y-intercept: none (0 not in domain); x-intercept: none, since $\sec x$ and $\tan x$ are both positive on the domain **C.** No symmetry **D.** $\lim_{x \to (\pi/2)^-} f(x) = \infty$, so $x = \frac{\pi}{2}$ is a VA.

E. $f'(x) = \sec x\ \tan x + \sec^2 x = \sec x\,(\tan x + \sec x) > 0$ on $\left(0, \frac{\pi}{2}\right)$, so f is increasing on $\left(0, \frac{\pi}{2}\right)$.

F. No local extrema

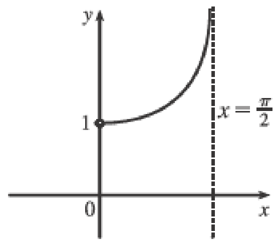
G. $f''(x) = \sec x\,(\sec^2 x + \sec x \tan x) + (\tan x + \sec x)\ \sec x\ \tan x$

$$= \sec x\,(\sec x + \tan x)\ \sec x + \sec x\,(\sec x + \tan x)\ \tan x$$

$$= \sec x\,(\sec x + \tan x)(\sec x + \tan x) = \sec x\,(\sec x + \tan x)^2 > 0$$

on $\left(0, \frac{\pi}{2}\right)$, so f is CU on $\left(0, \frac{\pi}{2}\right)$. No IP

H.



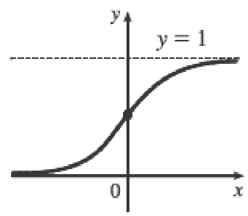
41. $y = 1/(1 + e^{-x})$ **A.** $D = \mathbb{R}$ **B.** No x-intercept; y-intercept $= f(0) = \frac{1}{2}$. **C.** No symmetry

D. $\lim_{x \to \infty} 1/(1 + e^{-x}) = \frac{1}{1+0} = 1$ and $\lim_{x \to -\infty} 1/(1 + e^{-x}) = 0$ since $\lim_{x \to -\infty} e^{-x} = \infty$], so f has horizontal asymptotes $y = 0$ and $y = 1$. **E.** $f'(x) = -(1 + e^{-x})^{-2}(-e^{-x}) = e^{-x}/(1 + e^{-x})^2$. This is positive for all x, so f is increasing on $\mathbb{R}$.

F. No extreme values **G.** $f''(x) = \dfrac{(1 + e^{-x})^2(-e^{-x}) - e^{-x}(2)(1 + e^{-x})(-e^{-x})}{(1 + e^{-x})^4} = \dfrac{e^{-x}(e^{-x} - 1)}{(1 + e^{-x})^3}$

The second factor in the numerator is negative for $x > 0$ and positive for $x < 0$, and the other factors are always positive, so f is CU on $(-\infty, 0)$ and CD on $(0, \infty)$. IP at $\left(0, \frac{1}{2}\right)$

H.



46. $y = f(x) = \ln(x^2 - 3x + 2) = \ln\left[(x - 1)(x - 2)\right]$ **A.** $D = \left\{x \text{ in } \mathbb{R}\colon x^2 - 3x + 2 > 0\right\} = (-\infty, 1) \cup (2, \infty)$.

B. y-intercept: $f(0) = \ln 2$; x-intercepts: $f(x) = 0 \iff x^2 - 3x + 2 = e^0 \iff x^2 - 3x + 1 = 0 \iff$ $x = \frac{3 \pm \sqrt{5}}{2} \Rightarrow x \approx 0.38, 2.62$ **C.** No symmetry **D.** $\lim_{x \to 1^-} f(x) = \lim_{x \to 2^+} f(x) = -\infty$, so $x = 1$ and $x = 2$ are VAs.

No HA **E.** $f'(x) = \dfrac{2x - 3}{x^2 - 3x + 2} = \dfrac{2(x - 3/2)}{(x - 1)(x - 2)}$, so $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 2$. Thus, f is decreasing on $(-\infty, 1)$ and increasing on $(2, \infty)$. **F.** No extreme values

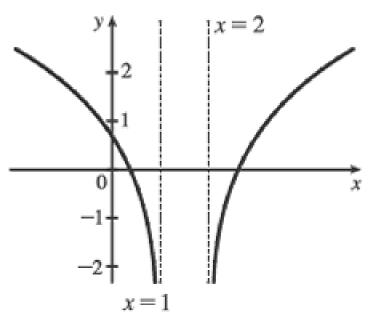
G. $f''(x) = \dfrac{(x^2 - 3x + 2) \cdot 2 - (2x - 3)^2}{(x^2 - 3x + 2)^2}$

$$= \frac{2x^2 - 6x + 4 - 4x^2 + 12x - 9}{(x^2 - 3x + 2)^2} = \frac{-2x^2 + 6x - 5}{(x^2 - 3x + 2)^2}$$

The numerator is negative for all x and the denominator is positive, so $f''(x) < 0$ for all x in the domain of f. Thus, f is CD on $(-\infty, 1)$ and $(2, \infty)$.

No IP

H.



51. $y = f(x) = e^{3x} + e^{-2x}$ **A.** $D = \mathbb{R}$ **B.** y-intercept $= f(0) = 2$; no x-intercept **C.** No symmetry **D.** No asymptote

E. $f'(x) = 3e^{3x} - 2e^{-2x}$, so $f'(x) > 0 \;\Leftrightarrow\; 3e^{3x} > 2e^{-2x}$ [multiply by e^{2x}] $\Leftrightarrow$ $e^{5x} > \frac{2}{3} \;\Leftrightarrow\; 5x > \ln\frac{2}{3} \;\Leftrightarrow\; x > \frac{1}{5}\ln\frac{2}{3} \approx -0.081$. Similarly, $f'(x) < 0 \;\Leftrightarrow$ $x < \frac{1}{5}\ln\frac{2}{3}$. f is decreasing on $\left(-\infty, \frac{1}{5}\ln\frac{2}{3}\right)$ and increasing on $\left(\frac{1}{5}\ln\frac{2}{3}, \infty\right)$.

F. Local minimum value $f\left(\frac{1}{5}\ln\frac{2}{3}\right) = \left(\frac{2}{3}\right)^{3/5} + \left(\frac{2}{3}\right)^{-2/5} \approx 1.96$; no local maximum.

G. $f''(x) = 9e^{3x} + 4e^{-2x}$, so $f''(x) > 0$ for all x, and f is CU on $(-\infty, \infty)$. No IP

H.