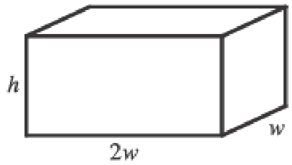
14.



$V = lwh \quad \Rightarrow \quad 10 = (2w)(w)h = 2w^2h$, so $h = 5/w^2$.

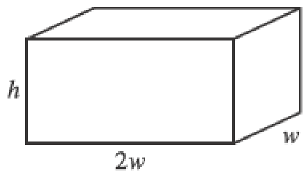
The cost is $10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh$, so

$C(w) = 20w^2 + 36w(5/w^2) = 20w^2 + 180/w$.

$C'(w) = 40w - 180/w^2 = 40\left(w^3 - \frac{9}{2}\right)/w^2 \quad \Rightarrow \quad w = \sqrt[3]{\frac{9}{2}}$ is the critical number. There is an absolute minimum for C when $w = \sqrt[3]{\frac{9}{2}}$ since $C'(w) < 0$ for $0 < w < \sqrt[3]{\frac{9}{2}}$ and $C'(w) > 0$ for $w > \sqrt[3]{\frac{9}{2}}$.

$C\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \dfrac{180}{\sqrt[3]{9/2}} \approx \$163.54$.

15.



$10 = (2w)(w)h = 2w^2h$, so $h = 5/w^2$. The cost is

$$C(w) = 10(2w^2) + 6[2(2wh) + 2hw] + 6(2w^2)$$
$$= 32w^2 + 36wh = 32w^2 + 180/w$$

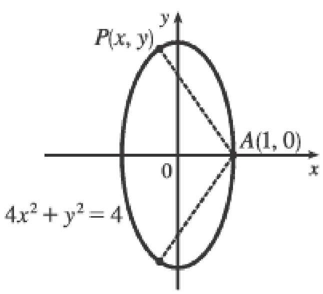
$C'(w) = 64w - 180/w^2 = 4(16w^3 - 45)/w^2 \quad \Rightarrow \quad w = \sqrt[3]{\frac{45}{16}}$ is the critical number. $C'(w) < 0$ for $0 < w < \sqrt[3]{\frac{45}{16}}$ and $C'(w) > 0$ for $w > \sqrt[3]{\frac{45}{16}}$. The minimum cost is $C\left(\sqrt[3]{\frac{45}{16}}\right) = 32(2.8125)^{2/3} + 180/\sqrt[3]{2.8125} \approx \$191.28$.

17. The distance from a point (x, y) on the line $y = 4x + 7$ to the origin is $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$. However, it is easier to work with the *square* of the distance; that is, $D(x) = \left(\sqrt{x^2 + y^2}\right)^2 = x^2 + y^2 = x^2 + (4x+7)^2$. Because the distance is positive, its minimum value will occur at the same point as the minimum value of D.

$D'(x) = 2x + 2(4x+7)(4) = 34x + 56$, so $D'(x) = 0 \quad \Leftrightarrow \quad x = -\frac{28}{17}$.

$D''(x) = 34 > 0$, so D is concave upward for all x. Thus, D has an absolute minimum at $x = -\frac{28}{17}$. The point closest to the origin is $(x, y) = \left(-\frac{28}{17}, 4\left(-\frac{28}{17}\right) + 7\right) = \left(-\frac{28}{17}, \frac{7}{17}\right)$.

19.



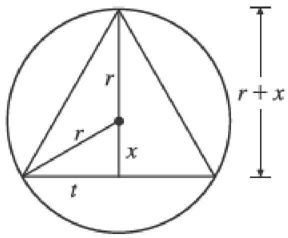
From the figure, we see that there are two points that are farthest away from $A(1, 0)$. The distance d from A to an arbitrary point $P(x, y)$ on the ellipse is $d = \sqrt{(x-1)^2 + (y-0)^2}$ and the square of the distance is

$S = d^2 = x^2 - 2x + 1 + y^2 = x^2 - 2x + 1 + (4 - 4x^2) = -3x^2 - 2x + 5$.

$S' = -6x - 2$ and $S' = 0 \quad \Rightarrow \quad x = -\frac{1}{3}$. Now $S'' = -6 < 0$, so we know that S has a maximum at $x = -\frac{1}{3}$. Since $-1 \le x \le 1$, $S(-1) = 4$, $S\left(-\frac{1}{3}\right) = \frac{16}{3}$, and $S(1) = 0$, we see that the maximum distance is $\sqrt{\frac{16}{3}}$. The corresponding y-values are

$y = \pm\sqrt{4 - 4\left(-\frac{1}{3}\right)^2} = \pm\sqrt{\frac{32}{9}} = \pm\frac{4}{3}\sqrt{2} \approx \pm 1.89$. The points are $\left(-\frac{1}{3}, \pm\frac{4}{3}\sqrt{2}\right)$.

25.



The area of the triangle is

$A(x) = \frac{1}{2}(2t)(r+x) = t(r+x) = \sqrt{r^2 - x^2}(r+x)$. Then

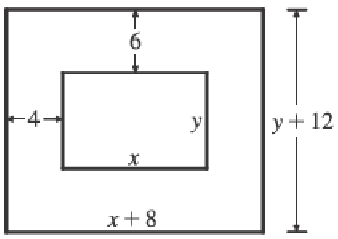
$$0 = A'(x) = r\frac{-2x}{2\sqrt{r^2-x^2}} + \sqrt{r^2-x^2} + x\frac{-2x}{2\sqrt{r^2-x^2}}$$

$$= -\frac{x^2+rx}{\sqrt{r^2-x^2}} + \sqrt{r^2-x^2} \quad \Rightarrow$$

$\frac{x^2+rx}{\sqrt{r^2-x^2}} = \sqrt{r^2-x^2} \quad \Rightarrow \quad x^2 + rx = r^2 - x^2 \quad \Rightarrow \quad 0 = 2x^2 + rx - r^2 = (2x-r)(x+r) \quad \Rightarrow$

$x = \frac{1}{2}r$ or $x = -r$. Now $A(r) = 0 = A(-r) \quad \Rightarrow$ the maximum occurs where $x = \frac{1}{2}r$, so the triangle has height $r + \frac{1}{2}r = \frac{3}{2}r$ and base $2\sqrt{r^2 - \left(\frac{1}{2}r\right)^2} = 2\sqrt{\frac{3}{4}r^2} = \sqrt{3}\,r$.

31.



$xy = 384 \quad \Rightarrow \quad y = 384/x$. Total area is

$A(x) = (8+x)(12+384/x) = 12(40 + x + 256/x)$, so

$A'(x) = 12(1 - 256/x^2) = 0 \quad \Rightarrow \quad x = 16$. There is an absolute minimum when $x = 16$ since $A'(x) < 0$ for $0 < x < 16$ and $A'(x) > 0$ for $x > 16$. When $x = 16$, $y = 384/16 = 24$, so the dimensions are 24 cm and 36 cm.

52. $y = 1 + 40x^3 - 3x^5 \quad \Rightarrow \quad y' = 120x^2 - 15x^4$, so the tangent line to the curve at $x = a$ has slope $m(a) = 120a^2 - 15a^4$. Now $m'(a) = 240a - 60a^3 = -60a(a^2 - 4) = -60a(a+2)(a-2)$, so $m'(a) > 0$ for $a < -2$, and $0 < a < 2$, and $m'(a) < 0$ for $-2 < a < 0$ and $a > 2$. Thus, m is increasing on $(-\infty, -2)$, decreasing on $(-2, 0)$, increasing on $(0, 2)$, and decreasing on $(2, \infty)$. Clearly, $m(a) \to -\infty$ as $a \to \pm\infty$, so the maximum value of $m(a)$ must be one of the two local maxima, $m(-2)$ or $m(2)$. But both $m(-2)$ and $m(2)$ equal $120 \cdot 2^2 - 15 \cdot 2^4 = 480 - 240 = 240$. So 240 is the largest slope, and it occurs at the points $(-2, -223)$ and $(2, 225)$. *Note:* $a = 0$ corresponds to a local *minimum* of m.