6. $f(x) = x(2-x)^2 = x(4-4x+x^2) = 4x - 4x^2 + x^3 \Rightarrow$

$F(x) = 4\left(\frac{1}{2}x^2\right) - 4\left(\frac{1}{3}x^3\right) + \frac{1}{4}x^4 + C = 2x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + C$

11. $f(x) = \dfrac{10}{x^9} = 10x^{-9}$ has domain $(-\infty, 0) \cup (0, \infty)$, so $F(x) = \begin{cases} \dfrac{10x^{-8}}{-8} + C_1 = -\dfrac{5}{4x^8} + C_1 & \text{if } x < 0 \\ -\dfrac{5}{4x^8} + C_2 & \text{if } x > 0 \end{cases}$

See Example 1(b) for a similar problem.

24. $f''(x) = 2 + x^3 + x^6 \Rightarrow f'(x) = 2x + \frac{1}{4}x^4 + \frac{1}{7}x^7 + C \Rightarrow f(x) = x^2 + \frac{1}{20}x^5 + \frac{1}{56}x^8 + Cx + D$

29. $f'(x) = 1 - 6x \Rightarrow f(x) = x - 3x^2 + C$. $f(0) = C$ and $f(0) = 8 \Rightarrow C = 8$, so $f(x) = x - 3x^2 + 8$.

34. $f'(x) = \dfrac{x^2-1}{x} = x - \dfrac{1}{x}$ has domain $(-\infty, 0) \cup (0, \infty) \Rightarrow f(x) = \begin{cases} \frac{1}{2}x^2 - \ln x + C_1 & \text{if } x > 0 \\ \frac{1}{2}x^2 - \ln(-x) + C_2 & \text{if } x < 0 \end{cases}$

$f(1) = \frac{1}{2} - \ln 1 + C_1 = \frac{1}{2} + C_1$ and $f(1) = \frac{1}{2} \Rightarrow C_1 = 0$.

$f(-1) = \frac{1}{2} - \ln 1 + C_2 = \frac{1}{2} + C_2$ and $f(-1) = 0 \Rightarrow C_2 = -\frac{1}{2}$.

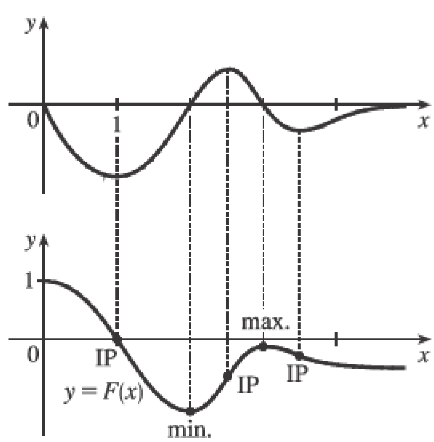
Thus, $f(x) = \begin{cases} \frac{1}{2}x^2 - \ln x & \text{if } x > 0 \\ \frac{1}{2}x^2 - \ln(-x) - \frac{1}{2} & \text{if } x < 0 \end{cases}$

39. $f''(\theta) = \sin\theta + \cos\theta \Rightarrow f'(\theta) = -\cos\theta + \sin\theta + C$. $f'(0) = -1 + C$ and $f'(0) = 4 \Rightarrow C = 5$, so $f'(\theta) = -\cos\theta + \sin\theta + 5$ and hence, $f(\theta) = -\sin\theta - \cos\theta + 5\theta + D$. $f(0) = -1 + D$ and $f(0) = 3 \Rightarrow D = 4$, so $f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4$.

49. b is the antiderivative of f. For small x, f is negative, so the graph of its antiderivative must be decreasing. But both a and c are increasing for small x, so only b can be f's antiderivative. Also, f is positive where b is increasing, which supports our conclusion.

50. We know right away that c cannot be f's antiderivative, since the slope of c is not zero at the x-value where $f = 0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f.

51.



The graph of F must start at $(0, 1)$. Where the given graph, $y = f(x)$, has a local minimum or maximum, the graph of F will have an inflection point.

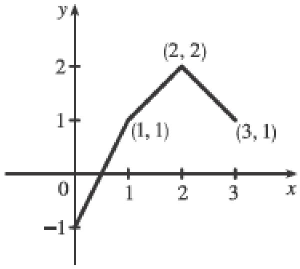
Where f is negative (positive), F is decreasing (increasing).

Where f changes from negative to positive, F will have a minimum.

Where f changes from positive to negative, F will have a maximum.

Where f is decreasing (increasing), F is concave downward (upward).

53.



$$f'(x) = \begin{cases} 2 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 < x < 2 \\ -1 & \text{if } 2 < x \le 3 \end{cases} \quad \Rightarrow \quad f(x) = \begin{cases} 2x + C & \text{if } 0 \le x < 1 \\ x + D & \text{if } 1 < x < 2 \\ -x + E & \text{if } 2 < x \le 3 \end{cases}$$

$f(0) = -1 \quad \Rightarrow \quad 2(0) + C = -1 \quad \Rightarrow \quad C = -1$. Starting at the point $(0, -1)$ and moving to the right on a line with slope 2 gets us to the point $(1, 1)$. The slope for $1 < x < 2$ is 1, so we get to the point $(2, 2)$. Here we have used the fact that f is continuous. We can include the point $x = 1$ on either the first or the second part of f. The line connecting $(1, 1)$ to $(2, 2)$ is $y = x$, so $D = 0$. The slope for $2 < x \le 3$ is -1, so we get to $(3, 1)$. $\quad f(3) = 1 \quad \Rightarrow \quad -3 + E = 1 \quad \Rightarrow \quad E = 4$. Thus

$$f(x) = \begin{cases} 2x - 1 & \text{if } 0 \le x \le 1 \\ x & \text{if } 1 < x < 2 \\ -x + 4 & \text{if } 2 \le x \le 3 \end{cases}$$

Note that $f'(x)$ does not exist at $x = 1$ or at $x = 2$.