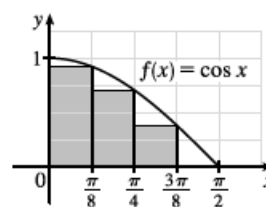
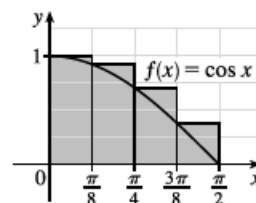


$$\begin{aligned}
 3. \text{ (a) } R_4 &= \sum_{i=1}^4 f(x_i) \Delta x \quad \left[\Delta x = \frac{\pi/2 - 0}{4} = \frac{\pi}{8} \right] = \left[\sum_{i=1}^4 f(x_i) \right] \Delta x \\
 &= [f(x_1) + f(x_2) + f(x_3) + f(x_4)] \Delta x \\
 &= \left[\cos \frac{\pi}{8} + \cos \frac{2\pi}{8} + \cos \frac{3\pi}{8} + \cos \frac{4\pi}{8} \right] \frac{\pi}{8} \\
 &\approx (0.9239 + 0.7071 + 0.3827 + 0) \frac{\pi}{8} \approx 0.7908
 \end{aligned}$$



Since f is decreasing on $[0, \pi/2]$, an *underestimate* is obtained by using the *right endpoint* approximation, R_4 .

$$\begin{aligned}
 \text{(b) } L_4 &= \sum_{i=1}^4 f(x_{i-1}) \Delta x = \left[\sum_{i=1}^4 f(x_{i-1}) \right] \Delta x \\
 &= [f(x_0) + f(x_1) + f(x_2) + f(x_3)] \Delta x \\
 &= \left[\cos 0 + \cos \frac{\pi}{8} + \cos \frac{2\pi}{8} + \cos \frac{3\pi}{8} \right] \frac{\pi}{8} \\
 &\approx (1 + 0.9239 + 0.7071 + 0.3827) \frac{\pi}{8} \approx 1.1835
 \end{aligned}$$



L_4 is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle; that is, $L_4 = R_4 + f(0) \cdot \frac{\pi}{8} - f(\frac{\pi}{2}) \cdot \frac{\pi}{8}$.

11. Since v is an increasing function, L_6 will give us a lower estimate and R_6 will give us an upper estimate.

$$L_6 = (0 \text{ ft/s})(0.5 \text{ s}) + (6.2)(0.5) + (10.8)(0.5) + (14.9)(0.5) + (18.1)(0.5) + (19.4)(0.5) = 0.5(69.4) = 34.7 \text{ ft}$$

$$R_6 = 0.5(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2) = 0.5(89.6) = 44.8 \text{ ft}$$

14. We can find an upper estimate by using the final velocity for each time interval. Thus, the distance d traveled after 62 seconds can be approximated by

$$d = \sum_{i=1}^6 v(t_i) \Delta t_i = (185 \text{ ft/s})(10 \text{ s}) + 319 \cdot 5 + 447 \cdot 5 + 742 \cdot 12 + 1325 \cdot 27 + 1445 \cdot 3 = 54,694 \text{ ft}$$

17. $f(x) = \sqrt[4]{x}$, $1 \leq x \leq 16$. $\Delta x = (16 - 1)/n = 15/n$ and $x_i = 1 + i \Delta x = 1 + 15i/n$.

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt[4]{1 + \frac{15i}{n}} \cdot \frac{15}{n}.$$

18. $f(x) = \frac{\ln x}{x}$, $3 \leq x \leq 10$. $\Delta x = (10 - 3)/n = 7/n$ and $x_i = 3 + i \Delta x = 3 + 7i/n$.

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\ln(3 + 7i/n)}{3 + 7i/n} \cdot \frac{7}{n}.$$