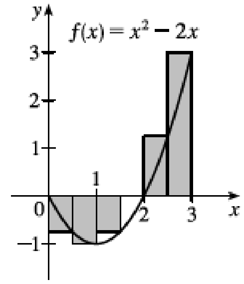
2. $f(x) = x^2 - 2x,\ 0 \le x \le 3.\quad \Delta x = \dfrac{b-a}{n} = \dfrac{3-0}{6} = \dfrac{1}{2}.$

Since we are using right endpoints, $x_i^* = x_i$.

$$\begin{aligned}R_6 &= \sum_{i=1}^{6} f(x_i)\,\Delta x \\ &= (\Delta x)\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)\right] \\ &= \tfrac{1}{2}\left[f\left(\tfrac{1}{2}\right) + f(1) + f\left(\tfrac{3}{2}\right) + f(2) + f\left(\tfrac{5}{2}\right) + f(3)\right] \\ &= \tfrac{1}{2}\left(-\tfrac{3}{4} - 1 - \tfrac{3}{4} + 0 + \tfrac{5}{4} + 3\right) = \tfrac{1}{2}\left(\tfrac{7}{4}\right) = \tfrac{7}{8}\end{aligned}$$

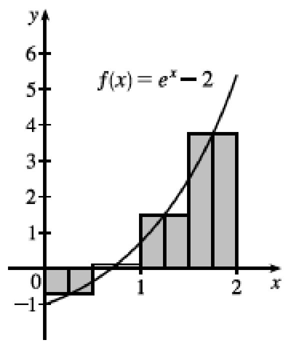


The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the three rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

3. $f(x) = e^x - 2,\ 0 \le x \le 2.\ \Delta x = \dfrac{b-a}{n} = \dfrac{2-0}{4} = \dfrac{1}{2}.$

Since we are using midpoints, $x_i^* = \overline{x}_i = \frac{1}{2}(x_{i-1} + x_i)$.

$$\begin{aligned}M_4 &= \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = (\Delta x)\left[f(\overline{x}_1) + f(\overline{x}_2) + f(\overline{x}_3) + f(\overline{x}_4)\right] \\ &= \tfrac{1}{2}\left[f\left(\tfrac{1}{4}\right) + f\left(\tfrac{3}{4}\right) + f\left(\tfrac{5}{4}\right) + f\left(\tfrac{7}{4}\right)\right] \\ &= \tfrac{1}{2}\left[(e^{1/4} - 2) + (e^{3/4} - 2) + (e^{5/4} - 2) + (e^{7/4} - 2)\right] \\ &\approx 2.322986\end{aligned}$$



The Riemann sum represents the sum of the areas of the three rectangles above the x-axis minus the area of the rectangle below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

6. (a) Using the right endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(x_i)\,\Delta x = 1[g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)] \approx 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5 = -0.5.$$

(b) Using the left endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(x_{i-1})\,\Delta x = 1[g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2)] \approx 2 + 1 - 0.5 - 1.5 - 1.5 - 0.5 = -1.$$

(c) Using the midpoint of each subinterval to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\begin{aligned}\sum_{i=1}^{6} g(\overline{x}_i)\,\Delta x &= 1[g(-2.5) + g(-1.5) + g(-0.5) + g(0.5) + g(1.5) + g(2.5)] \\ &\approx 1.5 + 0 - 1 - 1.75 - 1 + 0.5 = -1.75\end{aligned}$$

7. Since f is increasing, $L_5 \le \int_0^{25} f(x)\,dx \le R_5$.

$$\text{Lower estimate} = L_5 = \sum_{i=1}^{5} f(x_{i-1})\,\Delta x = 5[f(0)+f(5)+f(10)+f(15)+f(20)]$$
$$= 5(-42-37-25-6+15) = 5(-95) = -475$$

$$\text{Upper estimate} = R_5 = \sum_{i=1}^{5} f(x_i)\,\Delta x = 5[f(5)+f(10)+f(15)+f(20)+f(25)]$$
$$= 5(-37-25-6+15+36) = 5(-17) = -85$$

17. On $[2,6]$, $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} x_i \ln(1+x_i^2)\,\Delta x = \int_2^6 x\ln(1+x^2)\,dx$.

18. On $[\pi, 2\pi]$, $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} \dfrac{\cos x_i}{x_i}\,\Delta x = \displaystyle\int_\pi^{2\pi} \frac{\cos x}{x}\,dx$.

33. (a) Think of $\int_0^2 f(x)\,dx$ as the area of a trapezoid with bases 1 and 3 and height 2. The area of a trapezoid is $A = \frac{1}{2}(b+B)h$, so $\int_0^2 f(x)\,dx = \frac{1}{2}(1+3)2 = 4$.

(b) $$\int_0^5 f(x)\,dx = \underbrace{\int_0^2 f(x)\,dx}_{\text{trapezoid}} + \underbrace{\int_2^3 f(x)\,dx}_{\text{rectangle}} + \underbrace{\int_3^5 f(x)\,dx}_{\text{triangle}}$$
$$= \tfrac{1}{2}(1+3)2 + 3\cdot 1 + \tfrac{1}{2}\cdot 2\cdot 3 = 4+3+3 = 10$$

(c) $\int_5^7 f(x)\,dx$ is the negative of the area of the triangle with base 2 and height 3. $\int_5^7 f(x)\,dx = -\frac{1}{2}\cdot 2\cdot 3 = -3$.

(d) $\int_7^9 f(x)\,dx$ is the negative of the area of a trapezoid with bases 3 and 2 and height 2, so it equals $-\frac{1}{2}(B+b)h = -\frac{1}{2}(3+2)2 = -5$. Thus,

$$\int_0^9 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx + \int_7^9 f(x)\,dx = 10 + (-3) + (-5) = 2.$$

34. (a) $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ [area of a triangle]

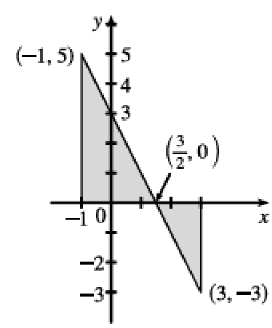
(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

(c) $\int_6^7 g(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ [area of a triangle]

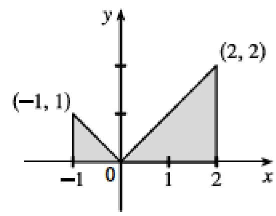
$$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \tfrac{1}{2} = 4.5 - 2\pi$$

38. $\int_{-1}^{3}(3-2x)\,dx$ can be interpreted as the area of the triangle above the x-axis minus the area of the triangle below the x-axis; that is,

$\frac{1}{2}\left(\frac{5}{2}\right)(5) - \frac{1}{2}\left(\frac{3}{2}\right)(3) = \frac{25}{4} - \frac{9}{4} = 4$.



39. $\int_{-1}^{2} |x|\, dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$.



47. $\int_{-2}^{2} f(x)\, dx + \int_{2}^{5} f(x)\, dx - \int_{-2}^{-1} f(x)\, dx = \int_{-2}^{5} f(x)\, dx + \int_{-1}^{-2} f(x)\, dx$ [by Property 5 and reversing limits]

$= \int_{-1}^{5} f(x)\, dx$ [Property 5]

48. $\int_{1}^{4} f(x)\, dx = \int_{1}^{5} f(x)\, dx - \int_{4}^{5} f(x)\, dx = 12 - 3.6 = 8.4$

49. $\int_{0}^{9} [2f(x) + 3g(x)]\, dx = 2\int_{0}^{9} f(x)\, dx + 3\int_{0}^{9} g(x)\, dx = 2(37) + 3(16) = 122$

50. If $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$, then $\int_{0}^{5} f(x)\, dx$ can be interpreted as the area of the shaded region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle whose legs have length 2. Thus, $\int_{0}^{5} f(x)\, dx = 5(3) + \frac{1}{2}(2)(2) = 17$.