

NAME: \_\_\_\_\_

Math 180

CALCULUS & ANALYTIC GEOMETRY  
EXAM # 1

Spring 2007

---

Read through the entire test before beginning. Make sure you have 10 (plus one bonus) questions on 6 pages. You may use your calculator and ask me questions if you find a problem unclear. Please be sure to show your work. *Unsupported answers will be counted as minimally correct.*

If you have time and the inclination, please consider filling out the *Reality Check*. I am asking you to reflect on how well you think you did on the exam. The student(s) who guess(es) closest to their actual score will be given a 2 point bonus. (If you know your score exactly, the bonus increases to 3 pts.)

Good luck and remember—you know quite a lot. Rely on your instincts and common sense. If something doesn't seem right, ASK! If you have no idea how to get started on a problem, ASK! If you are stuck, ASK! The worst thing that can happen is I look at you and say "You should know that."

---

Problem	Grade	Reality Check
1	/16	
2	/8	
3	/8	
4	/8	
5	/10	
6	/16	
7	/8	
8	/9	
9	/8	
10	/9	
*	/5	
Total	/100	

1. You are given the following information about two functions  $f$  and  $g$ :

$f$  is **odd**.

$g$  is **even**.

$x$	$f(x)$	$g(x)$
0	3	1
1	6	0
2	4	-3
3	8	-4
4	2	$\pi$

Calculate:

(a)  $f \cdot g(1)$  ( /4)

(b)  $f \circ g(1)$  ( /4)

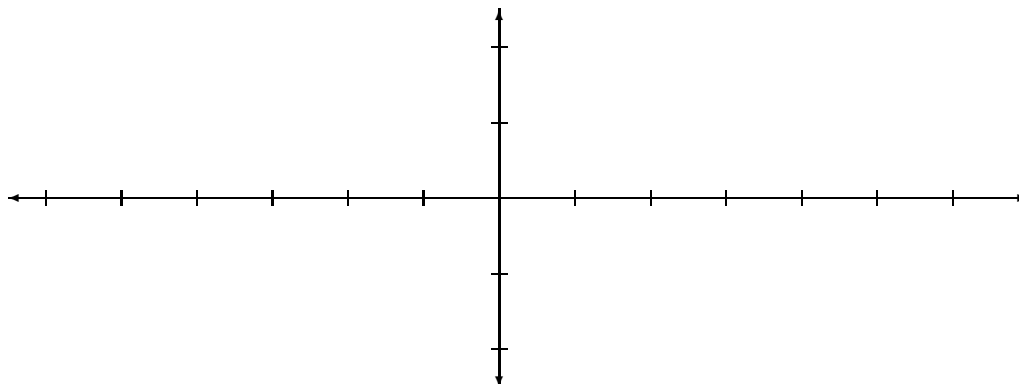
(c)  $f(-3)$  ( /4)

(d)  $g \circ g(2)$  ( /4)

2. If  $f(x) = x^2 - 2x + 3$ , evaluate and simplify the difference quotient  $\frac{f(a+h) - f(a)}{h}$ . ( /8)

3. Suppose  $g(x) = \begin{cases} \sin(x) & \text{for } x \leq 0 \\ x - 1 & \text{for } x > 0. \end{cases}$  Graph the function below.

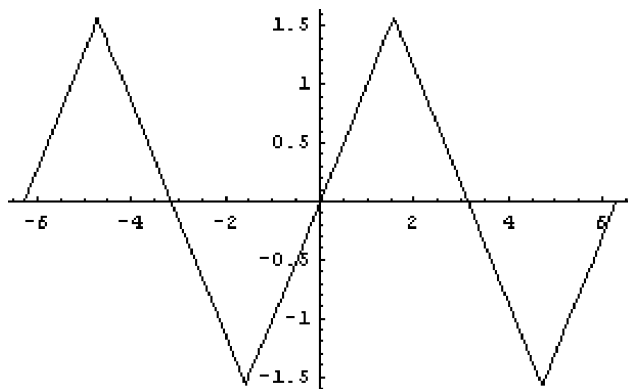
( /8)



4. Below is a sketch of the graph  $y = \sin^{-1}(\sin x)$ . Explain

- why one might expect the graph to be the line  $y = x$ ,
- why that expectation is incorrect, and
- why the given sketch is correct.

( /8)



5. Graph the two functions  $y = (e^x)^2$  and  $y = e^{(x^2)}$  and explain why they are different. What happens when you take the natural logarithm of both of these functions? ( /10)

6. Suppose that  $f(x)$  and  $g(x)$  are defined for all  $x$  and that  $\lim_{x \rightarrow \pi} f(x) = 4$  and  $\lim_{x \rightarrow \pi} g(x) = 13$ . Find the limits as  $x \rightarrow \pi$  of the following functions:

(a)  $f(x) + g(x)$  ( /4)

(b)  $f(x)/g(x)$  ( /4)

(c)  $x + g(x)$  ( /4)

(d)  $f(x) \cdot \frac{\sin(x - \pi)}{x - \pi}$  ( /4)

7. Find a function that satisfies the given conditions and sketch its graph

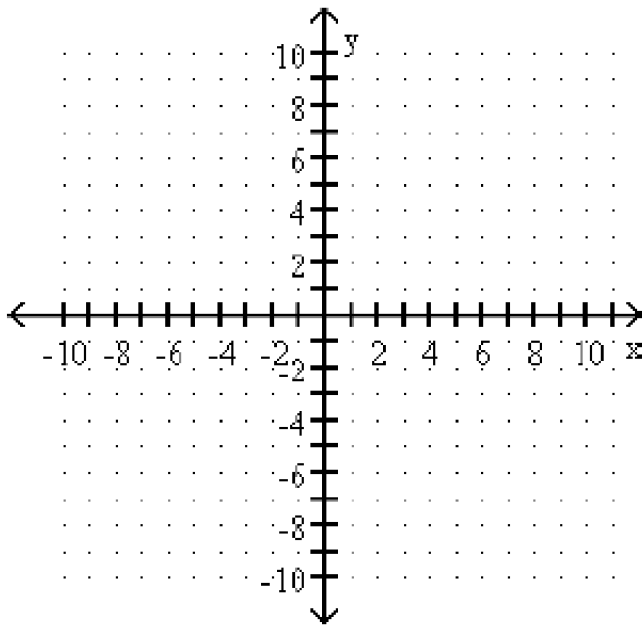
( /8)

$$\lim_{x \rightarrow -\infty} g(x) = -4$$

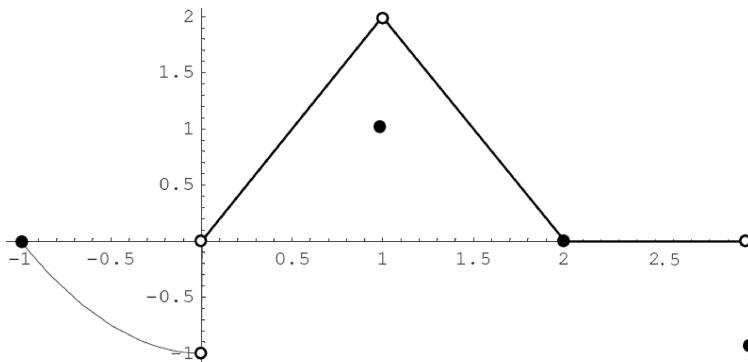
$$\lim_{x \rightarrow \infty} g(x) = 5$$

$$\lim_{x \rightarrow 0^+} g(x) = -6$$

$$\lim_{x \rightarrow 0^-} g(x) = 10$$



8. The graph of  $f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 \leq x < 3 \\ -1, & x = 3. \end{cases}$  is



State the values  $c$  in the interval  $[-1, 3]$  for which

(a)  $\lim_{x \rightarrow c^-} f(x)$  fails to exist.

( /3)

(b)  $f(x)$  is not continuous.

( /3)

(c)  $f(x)$  is not differentiable.

( /3)

9. If  $f(x) = 2x^3 - 5x + 5$ , show that there is at least one value of  $c$  for which  $f(x)$  equals  $\pi$ . ( /8)

10. **Numerical estimation of a derivative.** Use the information in the table below for the function  $g(x)$  to estimate the value of  $g'(2)$ . Explain your work. ( /9)

$x$	$g(x)$
1.5	.40547
1.9	.64185
1.95	.68813
1.999	.69265
2	.69315
2.001	.69365
2.05	.71784
2.1	.74194
1.5	.91629

\*. **Bonus Question.** Explain one mathematical topic that you studied to prepare for this examination but feel you did not get the opportunity to adequately show your knowledge. ( /5)