Calculus & Analytic Geometry I

Continuity

Did you consider this question?

It can be shown that the inequalities $1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2\cos x} < 1$ hold for all values of x close to 0. What does this tell you about $\lim_{x \to 0} \frac{x \sin x}{2 - 2\cos x}$?

Nice functions are continuous....

Definition. A function $f(x)$ is *continuous at a(n interior) point* c, if

$$\lim_{x \to c} f(x) = f(c).$$

A function is *continuous at a left endpoint* a or is *continuous at a right endpoint* b if

$$\lim_{x \to a^+} f(x) = f(a) \qquad \text{or} \qquad \lim_{x \to b^-} f(x) = f(b).$$

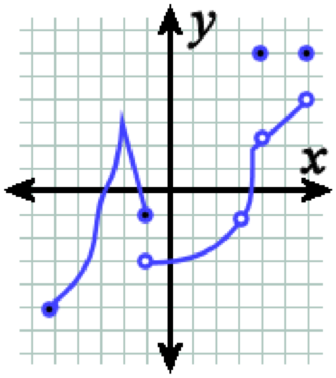
How can a function fail to be continuous at a point?

Definition. A function is called *continuous* if it is continuous at every point of its domain.

Is $f(x) = 1/x^2$ continuous?

Continuous functions include ... *Theorem 7, pg 124*

Where does the following function fail to be continuous on the interval $[-5, 6]$?



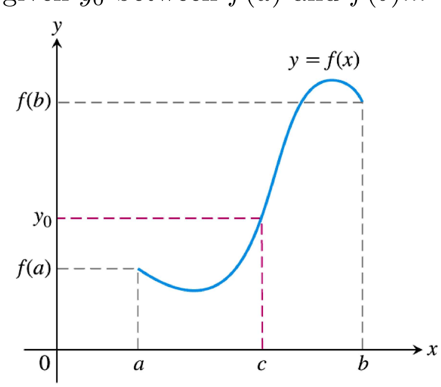
Good News—Algebraic combinations of continuous functions are continuous (whenever they are defined). So if f and g are continuous at $x = c$, then so are $f + g$, $f - g$, $f \cdot g$, $k \cdot f$ for a constant k, f/g provided $g(c) \neq 0$, and $f^{r/s}$ provided it is defined and r and s are integers. Inverses of continuous functions are continuous. Compositions of continuous functions are continuous.

Problems. For what real numbers do these functions fail to be continuous?

$$y = \frac{1}{x-2} - 3x \qquad\qquad y = \frac{1}{|x|+1} - \frac{x^2}{2}$$

$$y = \frac{\sin x}{x} \qquad\qquad y = \sqrt{2x+3}$$

Intermediate Value Theorem for Continuous Functions A function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. Said another way, given y_0 between $f(a)$ and $f(b)$...



Problem. Prove that there is a root of the equation $x^4 + x = 3$ in the interval $(1, 2)$.

Horizontal Asymptotes

Limits at positive or negative infinity tell us how the function "eventually" behaves (if ever)...

$f(x) = e^{-x}$ $\qquad g(t) = \dfrac{7t^3}{t^3 - 3t^2 + 6t}$ $\qquad h(z) = \dfrac{2 + \sqrt{z}}{2 - \sqrt{z}}$

$f(x) = \dfrac{2x^3 - 2x + 3}{3x^3 + 2x^2 - 5x}$ $\qquad h(x) = \dfrac{2x^2 - 2x + 3}{3x^3 + 2x^2 - 5x}$ $\qquad g(x) = \dfrac{2x^4 - 2x + 3}{3x^3 + 2x^2 - 5x}$

A line $y = b$ is a *horizontal asymptote* of the graph of a function $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = b \qquad \text{or} \qquad \lim_{x\to-\infty} f(x) = b.$$

Problem. Find the asymptotes (horizontal and vertical) for $f(x) = \dfrac{x^3 + 7x^2 - x - 7}{x^2 + x - 2}$.