Calculus & Analytic Geometry I

The Derivative: Analytic Viewpoint

Derivative of a Constant Function. For c a constant, the derivative of $f(x) = c$ equals

$f'(x) =$

Derivative of a Linear Function. If $f(x) = mx+b$, then $f'(x) =$ ____________________.

Derivative of a Constant Times a Function. If $f(x) = c \cdot g(x)$, then $f'(x) =$ ____________________.

Proof. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Derivatives of Sums and Differences. If $f(x) = g(x)+h(x)$, then $f'(x) =$ ____________________.

Proof. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

So if $f(x) = g(x) - h(x)$, then $f'(x) =$ ________________ because

Derivative of a Power Function. To calculate the derivative of x^2 (using the righthanded difference quotient) we had to multiply out $(x+h)^2$. In general, to find the derivative of x^n (for n and integer) we will have to multiply out $(x+h)^n$. Let's look at some examples:

$$\begin{aligned}
(x+h)^2 &= x^2 + 2xh + (h)^2 \\
(x+h)^3 &= x^3 + 3x^2h + 3x(h)^2 + (h)^3 \\
(x+h)^4 &= x^4 + 4x^3h + 6x^2(h)^2 + 4x(h)^3 + (h)^4 \\
&\vdots \\
(x+h)^n &= x^n + nx^{n-1}h + \underbrace{\cdots\cdots\cdots\cdots\cdots\cdots + (h)^n}_{\text{Terms involving } (h)^2 \text{ and higher powers of } h}
\end{aligned}$$

Now to find the derivative of $f(x) = x^n$:

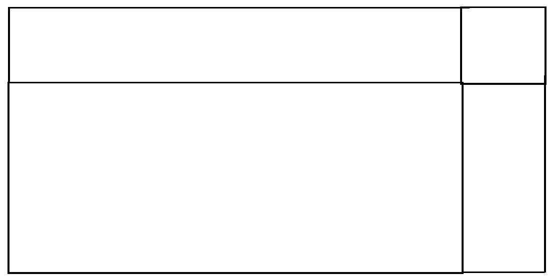
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} =$$

Derivative of an Exponential. If $f(x) = e^x$, then $f'(x) =$ ____________________.

Proof. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

Derivation of Product Rule. Suppose we know the derivatives of $f(x)$ and $g(x)$ and we want to calculate the derivative of the product $f(x)g(x)$.

$$(f(x)g(x))' = \lim_{h \to 0}$$



The Product Rule.

$$(fg)' = f'g + fg'$$

In words:

> The derivative of a product is the derivative of the first factor multiplied by the second, plus the first factor multiplied by the derivative of the second.

Quotient Rule. For completeness...Suppose we know the derivatives of $f(x)$ and $g(x)$ and we want to calculate the derivative of the quotient $f(x)/g(x)$.

$$(f/g)' = \frac{f'g - fg'}{g^2}.$$

We could derive it with difference quotients, but it will be much easier when we have the chain rule....

Apply your knowledge and find the following derivatives:

1. $y = (3 - x^2)(x^3 - x + 1)$
2. $v = \dfrac{1 + w - 4\sqrt{w}}{w}$
3. $z = \dfrac{12}{x^2}$