## The Derivative: Analytic Viewpoint

Recall the graphs of your favorite trig functions...


Now sketch their derivative functions based one where they are increasing, decreasing, or have a slope of zero.



So we conjecture that

$$
\frac{d}{d x} \sin x=\quad \frac{d}{d x} \cos x=
$$

Apply this knowledge.

1. Find $\frac{d}{d \theta}(\theta \sin \theta+\cos \theta)$.
2. Find $[(\sin t+\cos t)(\sin t-\cos t)]^{\prime}$.
3. Find the derivatives for the remaining trigonometric functions.
$(\tan x)^{\prime}=$
$(\sec x)^{\prime}=$
$(\cot x)^{\prime}=$
$(\csc x)^{\prime}=$

Last but not least, it is time to analytically verify that $\frac{d}{d x} \sin x=\cos x$ and $\frac{d}{d x} \cos x=-\sin x$. Recall the sum formulas:

$$
\sin (x+h)=\sin (x) \cos (h)+\sin (h) \cos (x) \quad \cos (x+h)=\cos (x) \cos (h)-\sin (x) \sin (h)
$$

$$
\sin ^{\prime}(x)=\lim _{h \rightarrow 0} \frac{\sin (x+h)-\sin (x-h)}{2 h}
$$

$$
\cos ^{\prime}(x)=\lim _{h \rightarrow 0} \frac{\cos (x+h)-\cos (x-h)}{2 h}
$$

