Derivatives of Inverse Functions

The chain rule is a powerful differentiation tool. It helps us determine slopes of

- composition of functions
  Find $\frac{d}{dx}\sec^2(x)$.

- parametric functions
  Suppose $x(t) = 2t^2 + 3$ and $y(t) = t^4$. Find $\frac{dy}{dx}$ at $t = -1$.

- implicitly defined functions
  Find $\frac{dr}{d\theta}$ if $e^{r^2\theta} = 2r + 2\theta$.

- inverse functions—as we shall see today.

Find the derivative of $\ln(x)$ by differentiating the identity
$e^{\ln(x)} = x$.

Problem. Find derivatives for the following functions:
$y = \ln(\sec x)$ $y = \ln[t(t+1)(t+2)(t+3)]$

Logarithmic differentiation. When you need to find a derivative involving lots of products or quotients, sometimes taking a logarithm first can help.

Find derivative of $y = \dfrac{\theta \sin\theta}{\sqrt{\sec\theta}}$.

Find the derivative of $f^{-1}(x)$ in terms of $f'(x)$ by differentiating the identity
$f(f^{-1}(x)) = x$.

Check yourself. Assume that $f(x)$ and $g(x)$ are inverse functions and

$$\begin{array}{lcrlcr} f(-2) & = & 1 & \qquad f'(-2) & = & 3 \\ f(1) & = & 7 & \qquad f'(1) & = & -10 \\ f(7) & = & -2 & \qquad f'(7) & = & -2 \end{array}$$

What is $g(7)$? Find $g'(-2)$.

Final Note. For $a > 0$ and u a differentiable function of x,
$\dfrac{d}{dx}a^u = a^u \ln a \dfrac{du}{dx}$ and $\dfrac{d}{dx}\log_a u = \dfrac{1}{u \ln a}\dfrac{du}{dx}$.