Calculus & Analytic Geometry I

Applications of Derivative to Other Disciplines

> Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them. —Joseph Fourier (1768-1830)

velocity	rate of change of displacement with respect to time
acceleration	rate of change of velocity with respect to time
linear density	rate of change of mass with respect to length
current	rate at which charge flows through a surface
rate of reaction	rate of change in concentration over time
compressibility	rate of change of volume with respect to pressure per unit volume
growth rate	rate of change of population over time
marginal cost	rate of change of cost with respect to the number of items produced
marginal revenue	rate of change of revenue with respect to the number of items produced
marginal profit	rate of change of profit with respect to number of items produced
sensitivity	rate of change of reaction with respect to the strength of some stimulus

1. **Profit Analysis.** A national toy distributor determines the cost and revenue models for one of its games.

$$C = 2.4x + .002x^2, \quad \text{for} \quad 0 \leq x \leq 6000$$
$$R = 7.2x - .001x^2, \quad \text{for} \quad 0 \leq x \leq 6000.$$

Determine the interval on which the profit function is increasing. Interpret the marginal profit when $x = 2000$.

2. **Lunar projectile motion.** A rock is thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of $s = 24t - 0.8t^2$ meters in t seconds.

   (a) Find the rock's velocity and acceleration at time t. (The acceleration in this case is the acceleration of gravity on the moon.)

   (b) How long does it take the rock to reach its highest point?

   (c) How high does the rock go?

   (d) How long does it take the rock to reach half its maximum height?

   (e) How long is the rock aloft?

3. **Learning Curve.** The rate at which a postal clerk can sort mail is a function of the clerk's experience. Suppose the postmaster of a large city estimates that after t months on the job, the average clerk can sort $Q(t) = 700 - 400e^{-.5t}$ letters per hour.

   (a) How many letters can a new employee sort per hour?

   (b) How many letters will a clerk with 6 months experience sort per hour?

   (c) Approximately how many letter will the average clerk ultimately be able to sort per hour?

Exponential Growth and Decay

When quantities grow (or decay) at a rate proportional to their size, we have

$$f'(t) = k \cdot f(t)$$

for some constant k that represents the *relative growth rate.* The only solution to this type of differential equation has the form

$$f(t) = Ce^{kt}.$$

Verify that $f(t) = Ce^{kt}$ **is a solution to** $f'(t) = k \cdot f(t)$.

1. **Growth.** A bacterial culture grows with constant relative growth rate. After 2 hours there are 600 bacteria and after 8 hours their count is 75,000.
   (a) Find the initial population.
   (b) Find an expression for the population after t hours.
   (c) Find the number of cells after 5 hours.
   (d) Find the rate of growth after 5 hours.
   (e) When will the population reach 200,000?
2. **Decay.** A sample of tritium-3 decayed to 94.5% of its original amount after a year.
   (a) What is the half-life of tritium-3?
   (b) How long would it take the sample to decay to 20% of its original amount?
3. **Newton's Law of Heating/Cooling**. The rate of heating/cooling of an object is proportional to the temperature difference between the object and its surroundings.

   $$\frac{dT}{dt} = k(T - T_s).$$

   Let $y = T - T_s$.

   A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°. If the temperature of the turkey is 150°F after 30 minutes, what is the temperature after 45 minutes? When will the turkey have cooled to 100°F?
4. **Interest.** If \$ 1000 is borrowed at 6.25% interest, find the amounts due at the end of 3 years if the interest is compounded (a) annually, (b) quarterly, (c) monthly, (d) weekly, (e) daily, (f) hourly, (g) continuously.