Calculus & Analytic Geometry I

Indeterminant Forms and l'Hôpital's Rule

Application of derivatives to assess pesky limits...

Indeterminant Forms. Sometimes we need to evaluate $\lim_{x\to c} \frac{f(x)}{g(x)}$ where $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ are either both 0 or both ∞ ($\frac{0}{0}$ or $\frac{\infty}{\infty}$).

Example. $\lim_{x\to 2} \frac{x^7 - 128}{x^3 - 8}$

l'Hôpital's Rule. Suppose that $f(c) = g(c) = 0$ and that f and g are differentiable on an open interval I containing c, and that $g'(x) \neq 0$ on I if $x \neq c$. Then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)},$$

assuming that the limit on the right side of this equation exists.

Problems. Verify the following limits:

1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$
2. $\lim_{x\to 0} \frac{x - \sin x}{x^3} = \frac{1}{6}$
3. $\lim_{x\to 0} \frac{1 - \cos x}{\sec x} = 0$
4. $\lim_{x\to \infty} \frac{2x^2 - 3x + 1}{3x^2 + 5x - 2} = \frac{2}{3}$
5. $\lim_{x\to \infty} \frac{x + \sin x}{x - \cos x} = 1$
6. $\lim_{x\to \infty} \left(1 + \frac{1}{x}\right)^x = e$
7. $\lim_{x\to 0+} x^{\sin x} = 1$

The Indeterminant Forms: $\frac{0}{0}$ $\frac{\infty}{\infty}$ $0 \cdot \infty$ $\infty - \infty$ 0^0 ∞^0 1^∞

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Curve Sketching

Problem. Sketch the general shape of a curve satisfying the given information

interval:	$x < 0$	$0 < x < 2$	$2 < x < 3$	$3 < x$
sign of f' :	$-$	$-$	$-$	$+$
sign of f'' :	$+$	$-$	$+$	$+$

Strategies for Graphing Functions

- Identify domain and any symmetries the curve may have.
- Find first and second derivatives.
- Find critical points and identify behavior at each.
- Determine where function is increasing or decreasing.
- Find points of inflection and concavity.
- Identify asymptotes (l'Hôpital may come in handy).
- Plot key points (intercepts and anything found above).

More Problems. Graph as many of the following functions as time will permit.

1. $y = 4x^3 - x^4$
2. $y = 2x - 3x^{2/3}$
3. $y = e^{2/x}$
4. $y = \dfrac{(x+1)^2}{1+x^2}$